

# **Welcome to GCSE 7+**

## **Monday 12 April 2021**

### **Session 6: Angles and Circles**

# A puzzle to ponder

place a 'plus'  
somewhere on  
the grid

$$38 \times 58 =$$

what happens  
when you  
multiply  
opposite  
numbers?

(2)



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

$$47 \times 46 =$$

# Keep GCSE 7+ safe for everyone

- **Do not ASK** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.
- **Do not GIVE** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.
- If **anyone** asks you, in the Chat or directly, for your personal contact details, or
- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or
- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**
  - email the Head teacher [dan.abramson@kcl.ac.uk](mailto:dan.abramson@kcl.ac.uk) or text him 07902 911144 and say what your concern is,
  - or email [kclmsoutreach@kcl.ac.uk](mailto:kclmsoutreach@kcl.ac.uk) and ask Dan to contact you.

# Right angled triangles



- You need to know that

Angle A	sin A $\div$	cos A $=$	tan A
0°	0	1	0
30°	$\frac{1}{2}$ $\div$	$\frac{\sqrt{3}}{2}$ $= \frac{1}{2} \times \frac{\sqrt{3}}{1}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$ $\div$	$\frac{1}{2}$ $= \frac{\sqrt{3}}{2} \times \frac{1}{1}$	$\sqrt{3}$
90°	1	0	undefined

Handwritten notes:

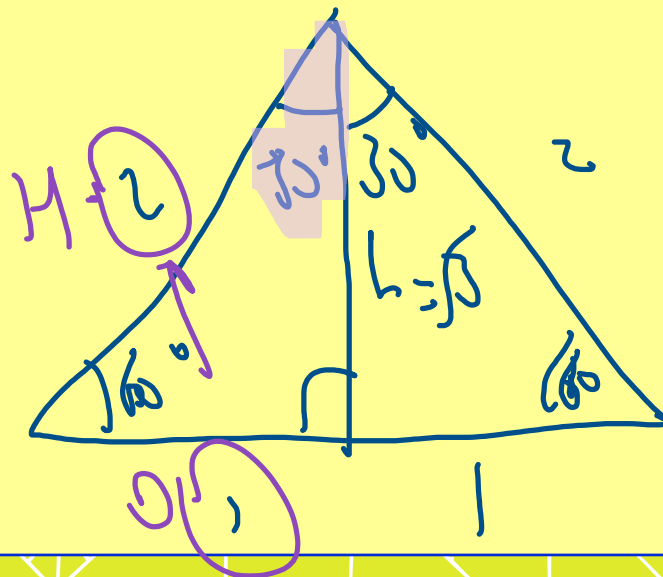
- A circled 'X' with a slash through it.
- A box containing the expression  $\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{1}$ .

# Right angled triangles



- You need to know why e.g.

Angle A	sin A	cos A	tan A
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$



$$h = \sqrt{2^2 - 1^2} \\ = \sqrt{3}$$

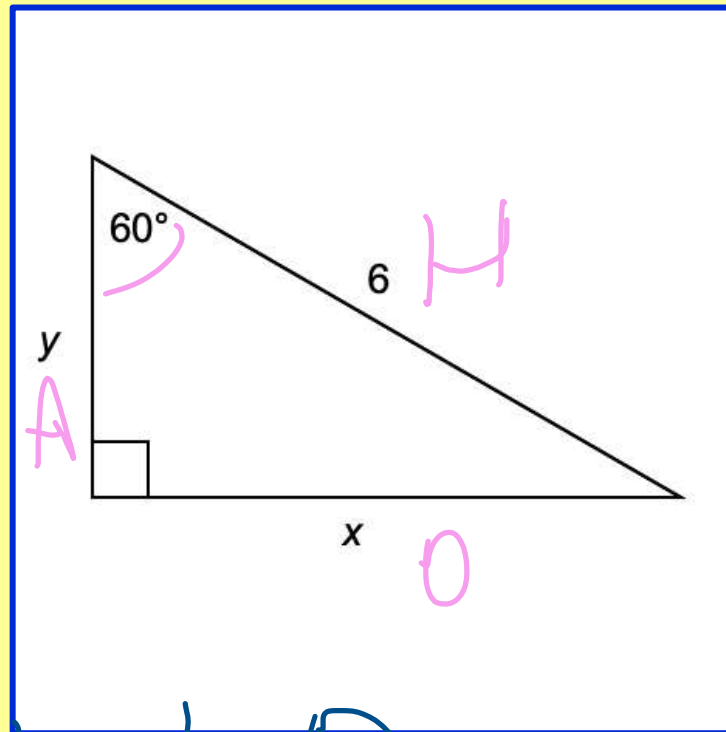
# Right angled triangles



- You need to know when to use these, e.g.

$$\begin{aligned}
 O &= (x)H \\
 &= \sin 60^\circ \times 6 \\
 &= \sqrt{3} \times 6 \\
 &= 3\sqrt{3} \\
 &= 5.196
 \end{aligned}$$

$$\begin{aligned}
 A &= C \times H \\
 &= 6 \times 6 \times \frac{1}{2} = 18
 \end{aligned}$$



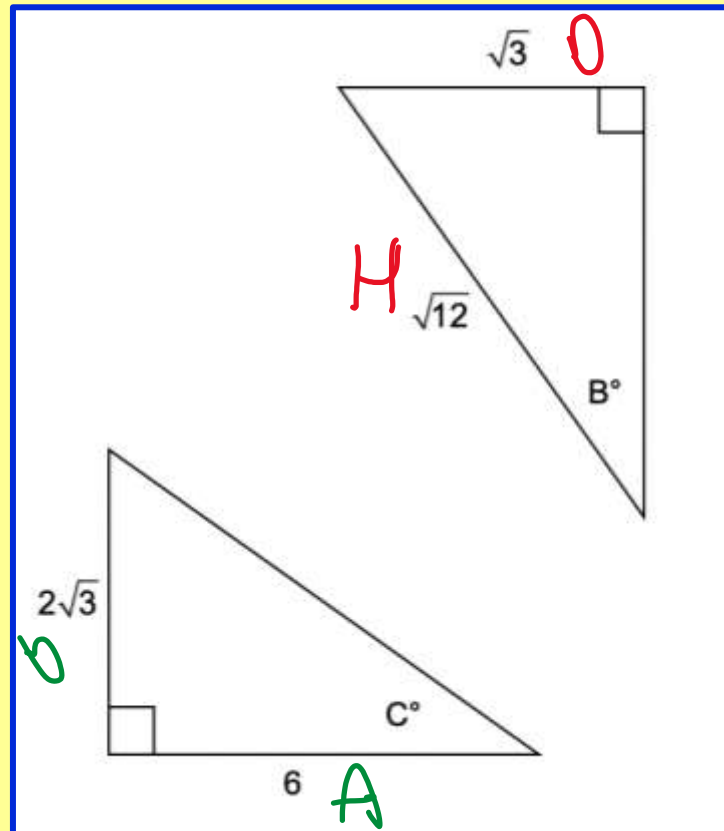
# Right angled triangles



- You need to know when to use these, e.g.

$$\tan C = \frac{\sqrt{3}}{3}$$

$$\Rightarrow C = 30^\circ$$

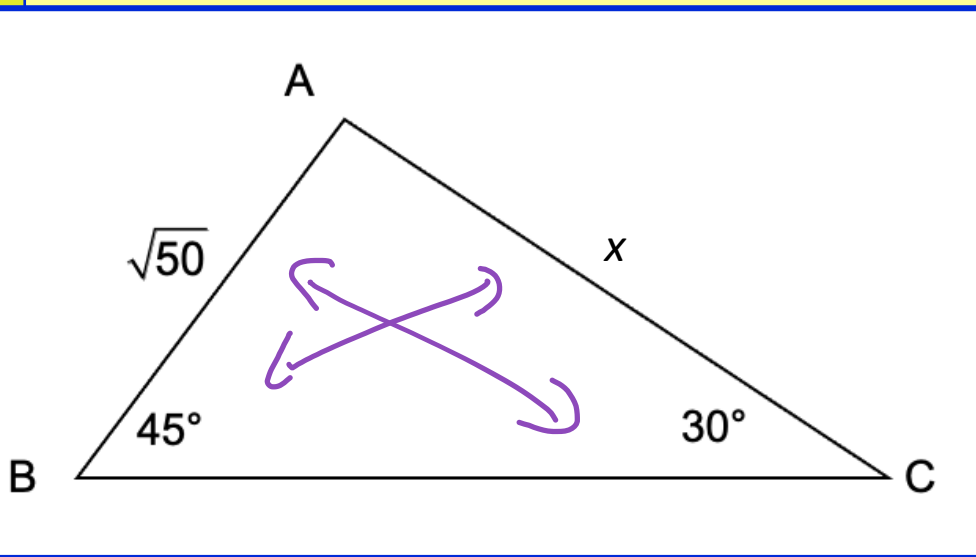


$$\begin{aligned} \sin B &= \frac{1}{\sqrt{12}} \\ &= \frac{1}{2\sqrt{3}} \\ &= \frac{1}{2} \\ \Rightarrow B &= 30^\circ \end{aligned}$$

# Right angled? triangles



- You need to know when to use these, e.g.



$$\frac{x}{\sin 45^\circ} = \frac{\sqrt{50}}{\sin 30^\circ}$$

$$\Rightarrow x \div \frac{1}{\sqrt{2}} = \sqrt{50} \div \frac{1}{2}$$

$$\Rightarrow x \times \frac{\sqrt{2}}{1} = \sqrt{50} \times \frac{2}{1}$$

$$\Rightarrow x \sqrt{2} = 2\sqrt{50} \Rightarrow x = \frac{2\sqrt{50}}{\sqrt{2}} = 2\sqrt{\frac{50}{2}} = 2\sqrt{25} = \boxed{10}$$

# Right angled? triangles



- You ought to know why we say that

Angle A	sin A	cos A	tan A
$90^\circ$	1	0	undefined



$$A \rightarrow 90$$

$$\sin A = \frac{0}{1} \Rightarrow \frac{\text{"H"}}{\text{"H"}} = 1$$

# Right angled? triangles

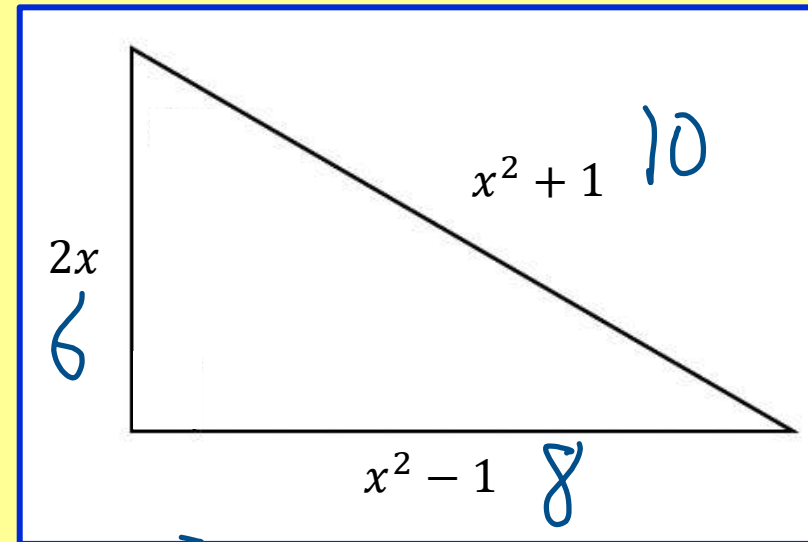


- Is this triangle right-angled?

$$x=3$$

$$\begin{aligned}
 & (2x)^2 + (x^2 - 1)^2 \\
 &= 4x^2 + x^4 - 2x^2 + 1 \\
 &= x^4 + 2x^2 + 1 \\
 &= (x^2 + 1)^2 = \text{hyp}^2
 \end{aligned}$$

Handwritten note: A small grid with a 2x2 layout. The top row contains 'x' and '1', and the bottom row contains '-1' and '1'. The four quadrants are each marked with a checkmark (✓).



$$\begin{aligned}
 6^2 + 8^2 &= 36 + 64 = 100 \\
 &= 10^2 \checkmark
 \end{aligned}$$

# Other triangles

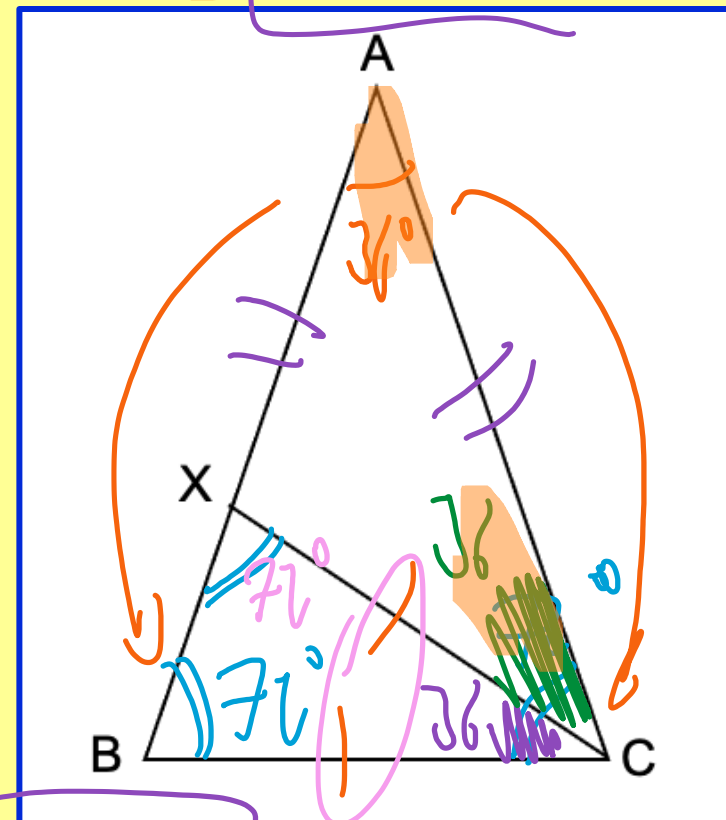


- $AB = AC$  and  $CX = CB$ . Angle  $BAC = 36^\circ$ . Prove that  $CX = XA$ .

$$\begin{aligned} \angle XC &= 72^\circ \\ \Rightarrow \angle XCB &= 144 - 72 - 72 \\ &= 36^\circ \end{aligned}$$

$$\angle XA = 36^\circ$$

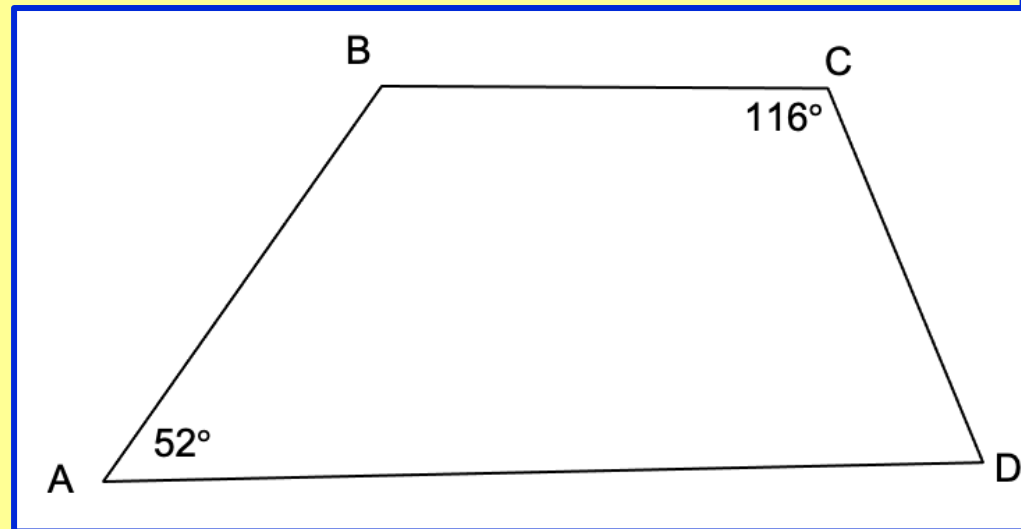
$$\Rightarrow \triangle AXC \text{ is isosceles} \Rightarrow AX = XC$$



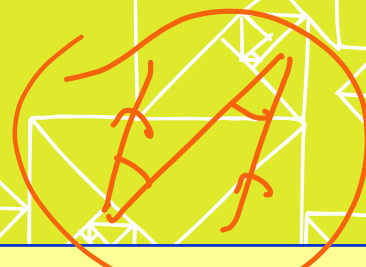
# Quadrilaterals



- Angle ABC is exactly twice angle ADC. What is the quadrilateral?



# Quadrilaterals



- ABCD is a rhombus and AP is the bisector of angle DAC. Prove that angle DPA is three times the size of angle DAP.

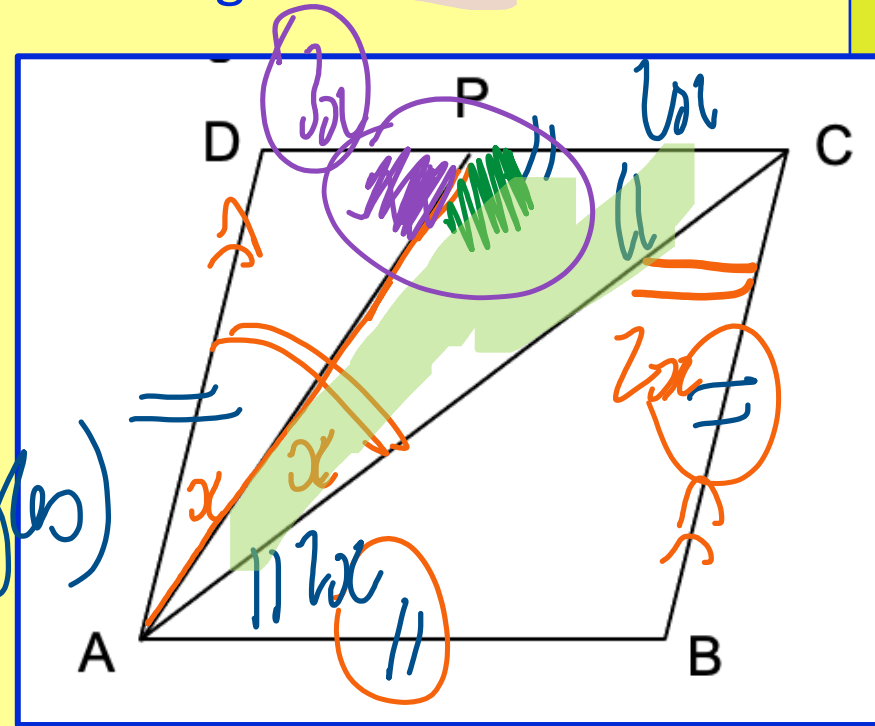
Let  $\hat{DAP} = x$

$\hat{PAC} = x$  also

$\Rightarrow \hat{ACD} = 2x$  (alternate angles)

$\angle DPA = 180 - 3x$

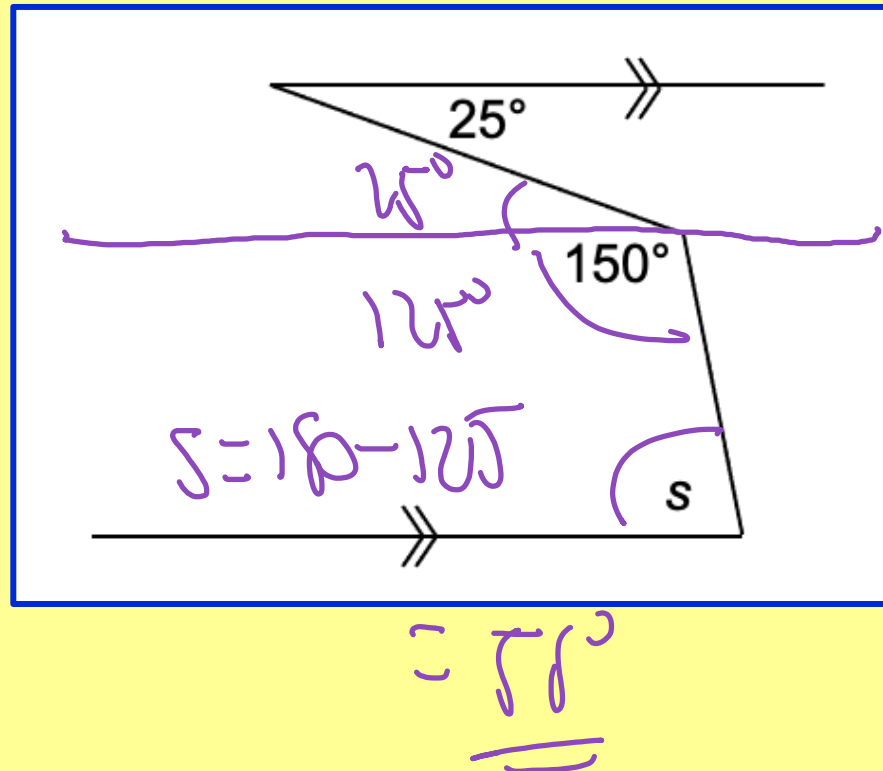
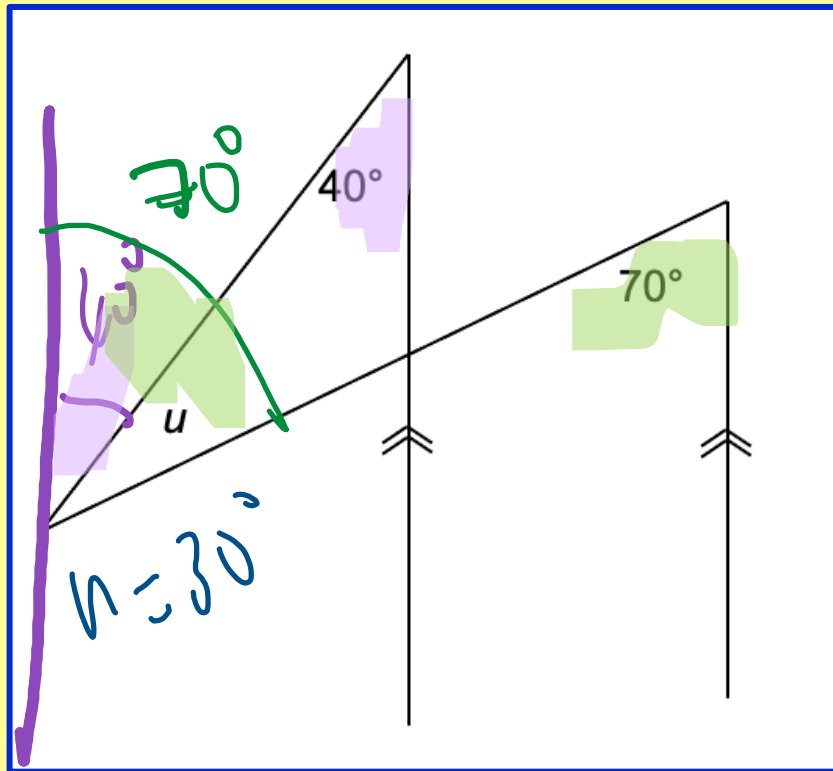
$\hat{DPA} = 3x$



# Parallel lines



- Work out the unknown angles

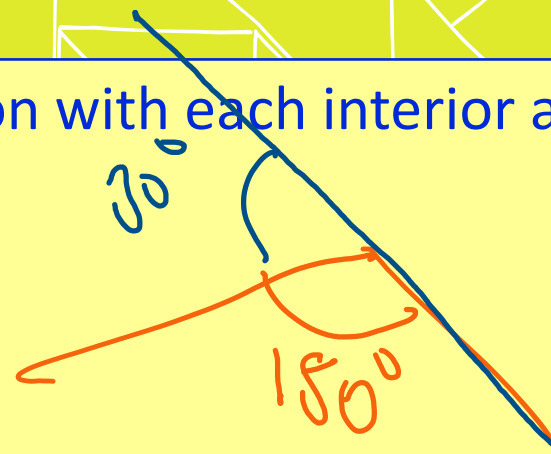


# Polygons

$$n \times 180 = 360$$

- How many sides has a regular polygon with each interior angle equal to  $150^\circ$ ?

$$n = \frac{360}{30} = 12$$



- Is there a regular polygon with each interior angle equal to

a)  $170^\circ$   $10^\circ$

$$360 \div 10 \checkmark$$

36-gon

b)  $173^\circ$   $7^\circ$

$$360 \div 7 \times$$

c)  $177^\circ$   $3^\circ$

$$360 \div 3 \checkmark$$

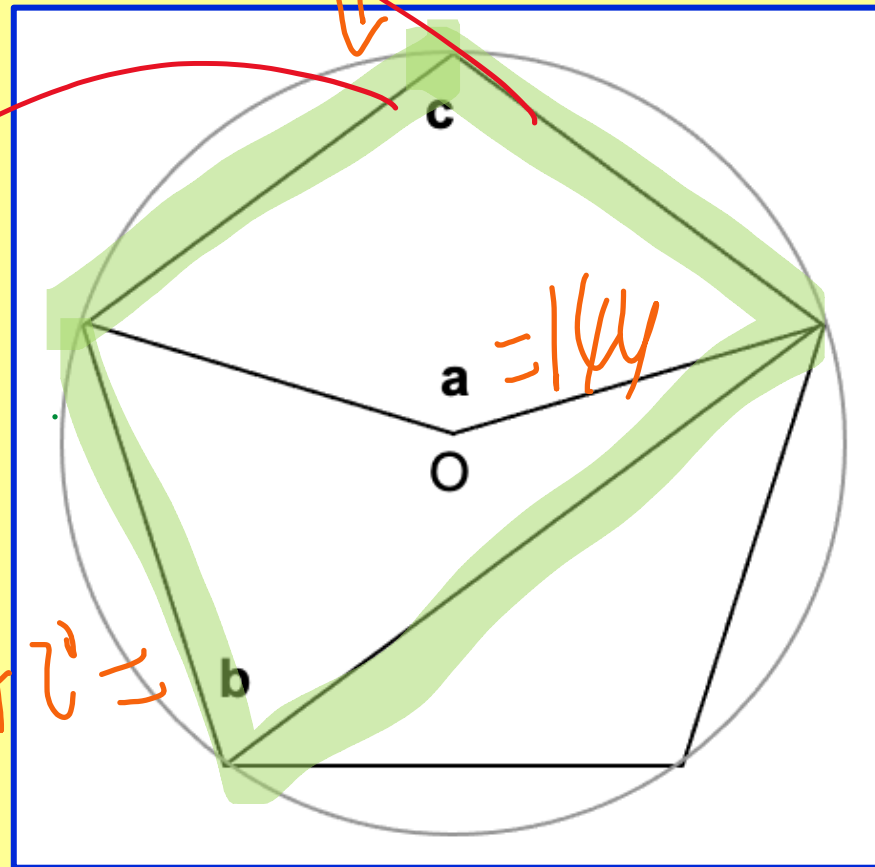
120-gon

ext

# Angles and Circles



- Work out the missing angles in this regular pentagon



$$\begin{aligned} 180 \\ - 72 \\ \hline = 108^\circ \end{aligned}$$

$$\begin{aligned} 180 - 108 \\ \hline = 72^\circ \end{aligned}$$

$$72^\circ =$$

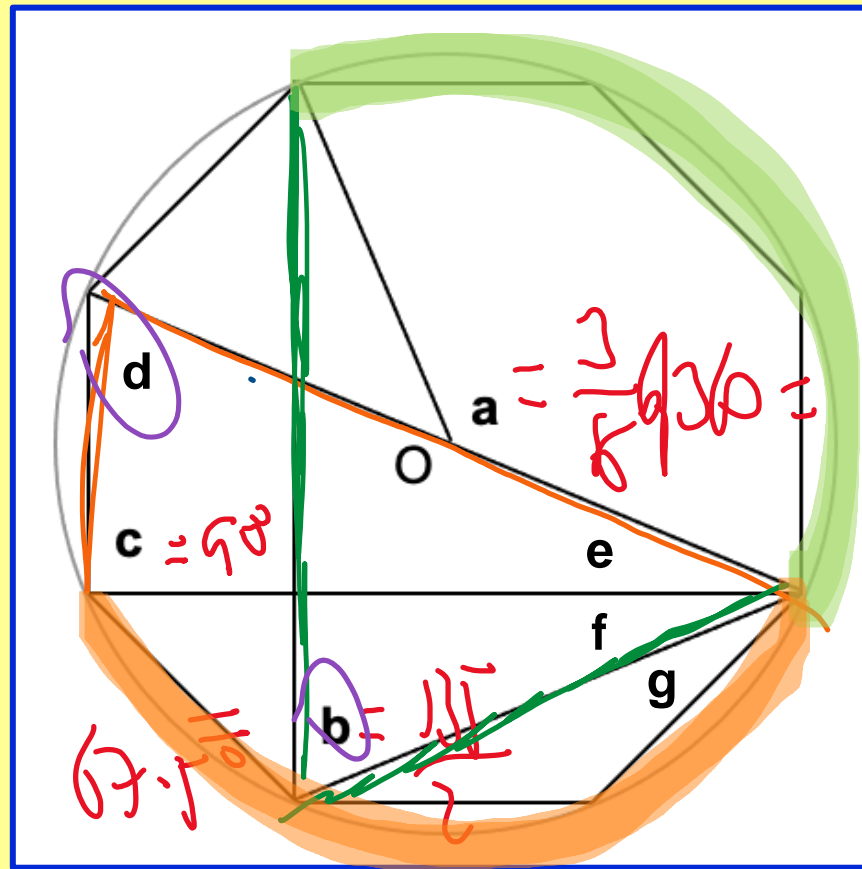
$$\begin{aligned} a &= 144^\circ \end{aligned}$$

$$\begin{aligned} n &= \frac{2}{5} \times 360 \\ &= 144^\circ \end{aligned}$$

# Angles and Circles



- Work out the missing angles in this regular octagon



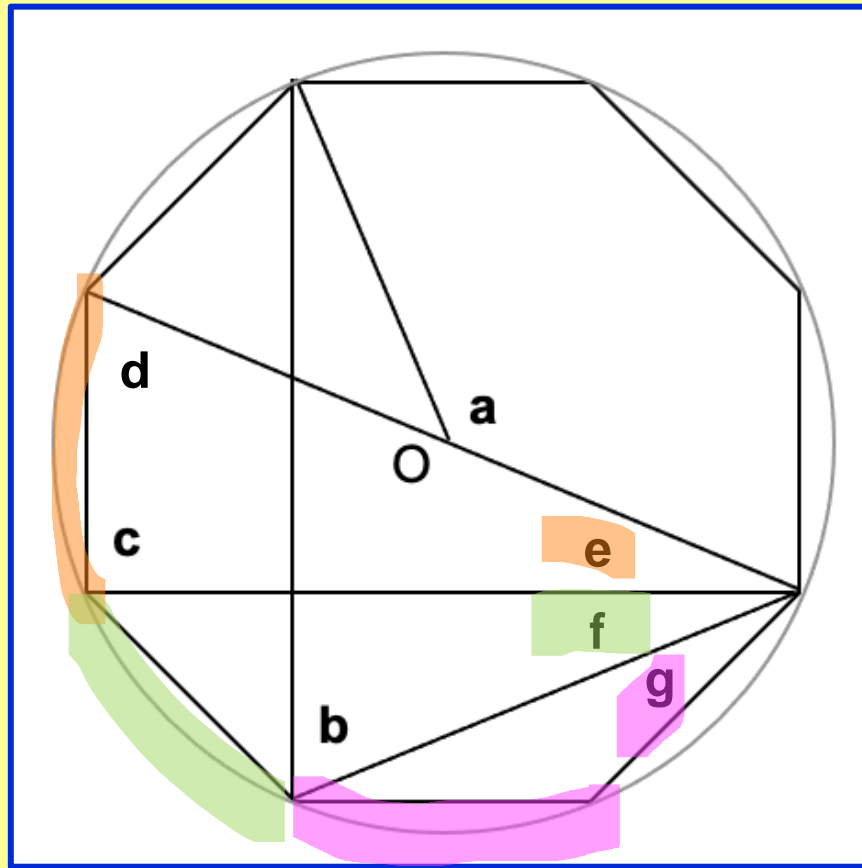
$$67.5 =$$

$$d = b \checkmark$$

# Angles and Circles



- Work out the missing angles in this regular octagon



$$e = f = g$$

# Angles and Circles



- Are the lines PT and PS tangents to the circle?

No

$$62 + 62 = 124$$
$$124 + 56 = 180$$

Alternate  
Segment  
Theorem

