

GCSE 7+ Session 6
Independent Practice
Angles and Circles



Revise, refresh, recall the core knowledge and skills:

1 Copy and complete this table, filling in the empty cells.

Name of regular polygon	Number of sides	Size of each exterior angle	Size of each interior angle
Pentagon	5	72°	108°
Hexagon	6	60°	120°
Octagon	8	45°	135°
Decagon	10	36°	144°
Dodecagon	12	30°	150°
24-gon	24	15°	165°
30-gon	30	12°	168°
90-gon	90	4°	176°
360-gon	360	1°	179°

Agree or challenge:

Challenge

a) the size of each interior angle of a regular polygon is directly proportional to the number of sides of the polygon

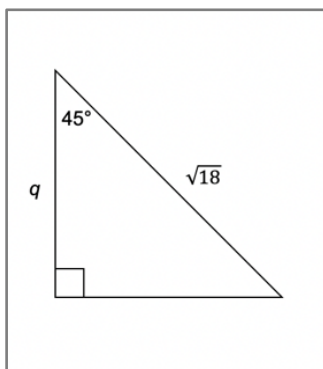
b) the size of each exterior angle of a regular polygon is inversely proportional to the number of sides of the polygon

Agree

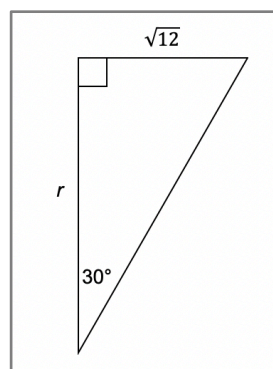
$$\text{ext} = 360 \div n$$



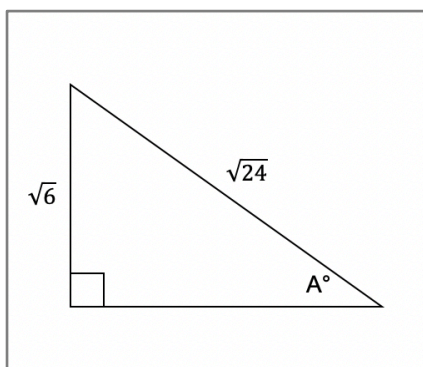
2 Work out the unknown sides or angles in these right-angled triangles:



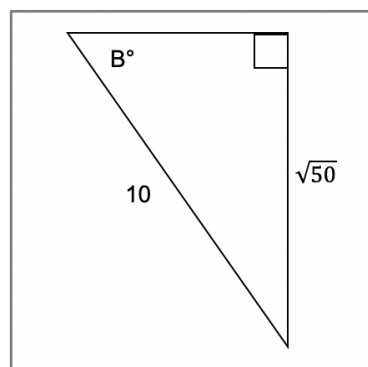
$$\begin{aligned} \cos 45^\circ &= \frac{q}{\sqrt{18}} \\ q &= \sqrt{18} \times \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{18}}{\sqrt{2}} = 3 \end{aligned}$$



$$\begin{aligned} \tan 30^\circ &= \frac{\sqrt{12}}{r} \\ r &= \frac{\sqrt{12}}{\tan 30^\circ} \\ &= \frac{\sqrt{12}}{\frac{1}{\sqrt{3}}} \\ &= \sqrt{12} \times \sqrt{3} = \sqrt{36} = 6 \end{aligned}$$

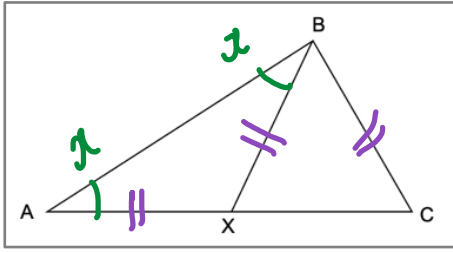


$$\begin{aligned} \sin A &= \frac{\sqrt{6}}{\sqrt{24}} \\ &= \frac{\sqrt{6}}{\sqrt{4 \times 6}} = \frac{\sqrt{6}}{2\sqrt{6}} = \frac{1}{2} \\ \therefore A &= 30^\circ \end{aligned}$$



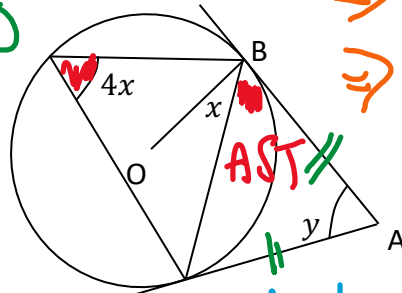
$$\begin{aligned} \sin B &= \frac{\sqrt{50}}{10} \\ &= \frac{\sqrt{2 \times 25}}{10} = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2} \\ \therefore B &= 45^\circ \end{aligned}$$

- 3 $AX = BX = BC$.
Prove that angle BCX is twice angle ABX .



Let $\hat{A}BX = x$
 $\Rightarrow \hat{X}AB = x$ (isosceles Δ)
 $\Rightarrow \hat{A}XB = 180 - 2x$
 $\Rightarrow \hat{B}XC = 2x$
 $\Rightarrow \hat{B}CX = 2x$ (isosceles Δ)
 $\Rightarrow \hat{B}CX$ is twice $\hat{A}BX$ \square

- 5 Work out x and y .
 O is the centre of the circle.
 AB and AC are tangents.

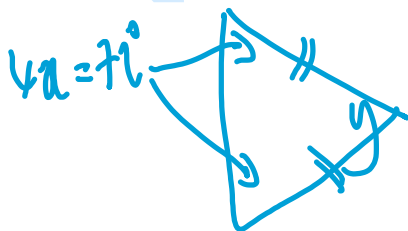


$\hat{A}BC = 90 - x$ (tangent perpendicular to radius)
 $\hat{A}BC$ also $= 4x$ (Alternate segment theorem)

$$\Rightarrow 90 - x = 4x$$

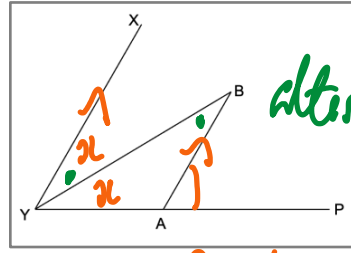
$$\Rightarrow 90 = 5x$$

$$\Rightarrow x = 18^\circ$$



$$y = 180 - 72 - 72 = 36^\circ$$

- 4 XY is parallel to BA . YB bisects angle XYP .
Prove that $AY = AB$.



alternate angles
 $\hat{X}YB = \hat{B}YA$ (YB is bisector)
Let $\hat{X}YB = x$
 $\Rightarrow \hat{Y}BA = x$ (alternate angle with $\hat{X}YB$)
 $\Rightarrow \hat{B}YA = \hat{Y}BA$
 \Rightarrow isosceles Δ
 $\Rightarrow AY = AB$ \square

\Rightarrow isosceles Δ

$\Rightarrow AY = AB$ \square

isosceles