Welcome to GCSE 7+
Monday 12 April 2021

Session 5:
Lines and Circles

A puzzle to ponder

How many **different** factorisations are there

- of the form $(ax + b)(cx + d)$
- where a, b, c, d are all integers

that, when expanded, simplify to a quadratic expression of the form

a) $x^2 + Mx + 8$
b) $15x^2 + Nx + 16$
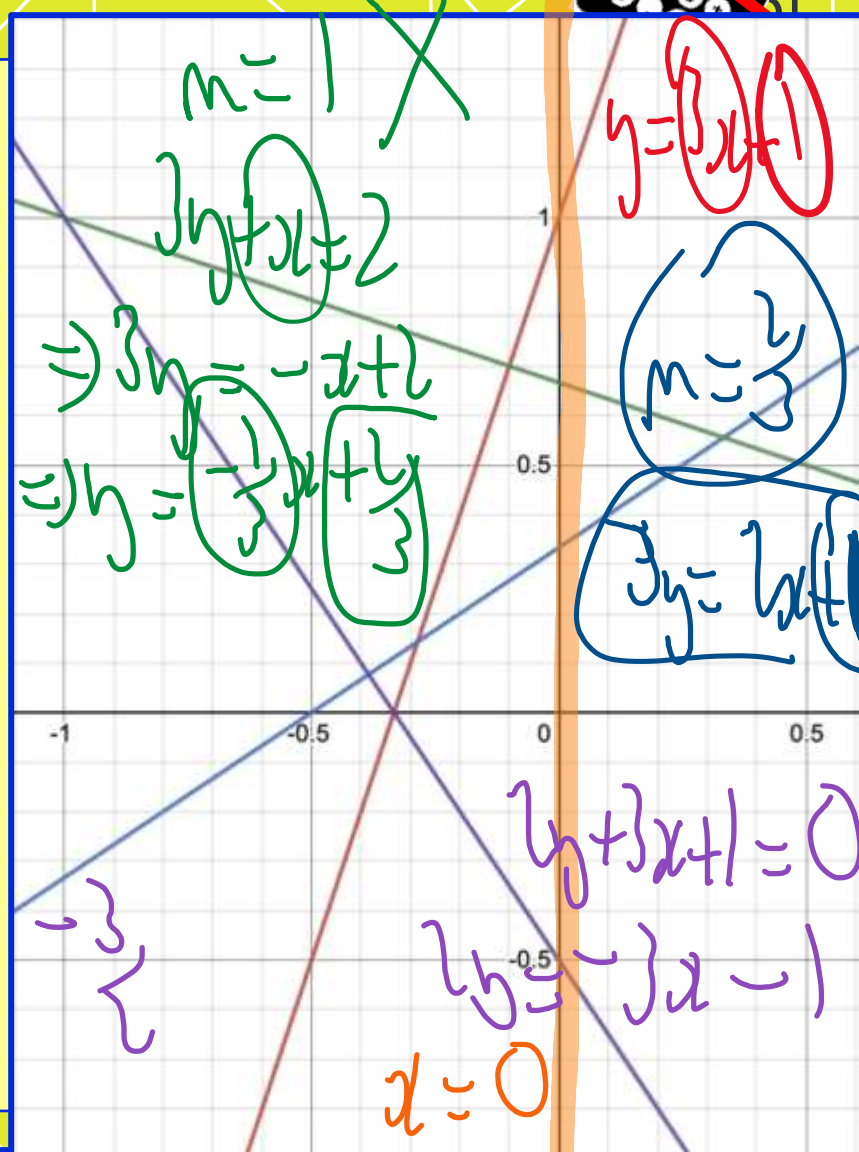
Keep GCSE 7+ safe for everyone

- **Do not <u>ASK</u>** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.

- **Do not <u>GIVE</u>** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.

- If **anyone** asks you, in the Chat or directly, for your personal contact details, or

- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or

- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**

  - email the Head teacher dan.abramson@kcl.ac.uk or text him 07902 911144 and say what your concern is,

  - or email kclmsoutreach@kcl.ac.uk and ask Dan to contact you.

Linear graphs

Match the linear equations and the graphs:

- $y = 3x + 1$ $(0, 1)$ $m = 3$

- $3y = 2x + 1$ $(0, \frac{1}{3})$ $m = \frac{2}{3}$

- $3y + x = 2$ $(0, \frac{2}{3})$ $m = \frac{2}{3}$

- $2y + 3x + 1 = 0$ $(0, -\frac{1}{2})$ $m = -3$

$m = 1$ X

$3y + x = 2$
$\Rightarrow 3y = -x + 2$
$\Rightarrow y = -\frac{1}{3}x + \frac{2}{3}$

$y = 3x + 1$

$m = \frac{2}{3}$

$3y = 2x + 1$

$2y + 3x + 1 = 0$

$2y = -3x - 1$

$x = 0$

Linear graphs

Match the linear equations and the graphs:

gradient	y-intercept

- $y = 3x + 1$

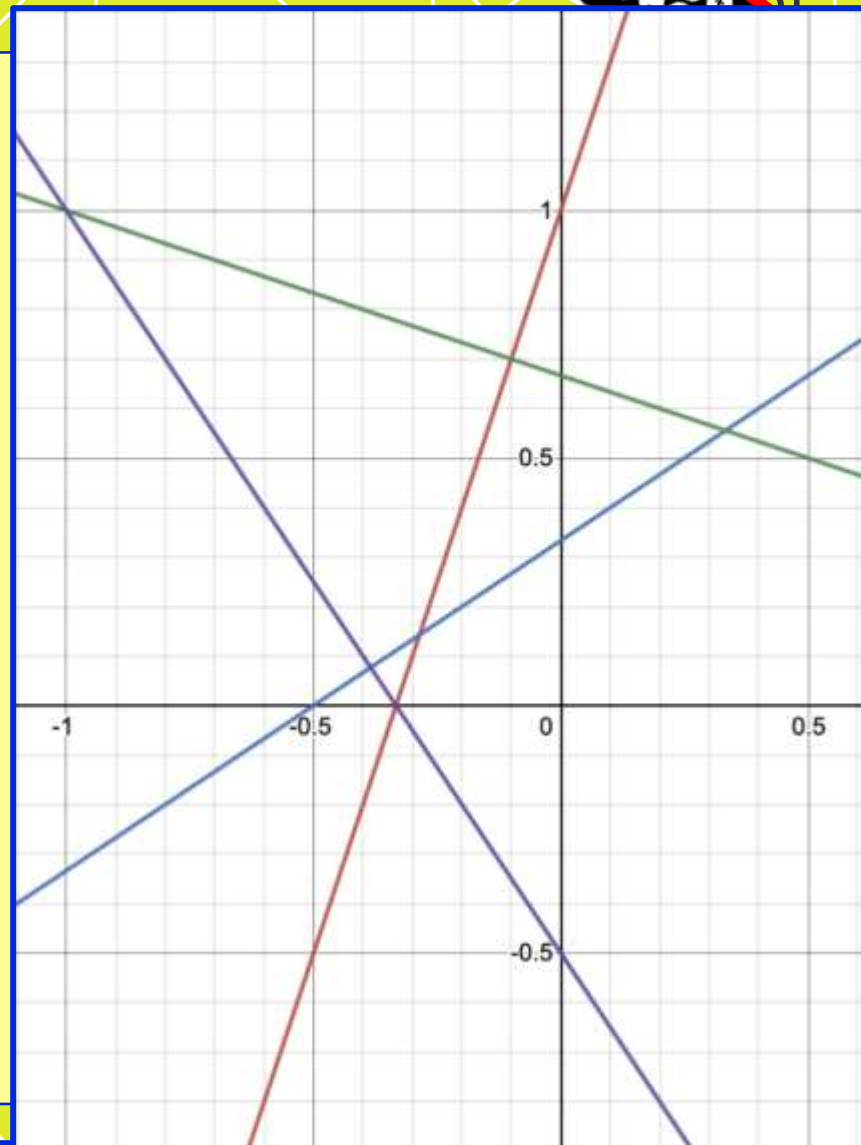- $3y = 2x + 1$

- $3y + x = 2$

- $2y + 3x + 1 = 0$

Linear graphs

Match the linear equations and the graphs:

- $y = 3x + 1$

- $3y = 2x + 1$

- $3y + x = 2$
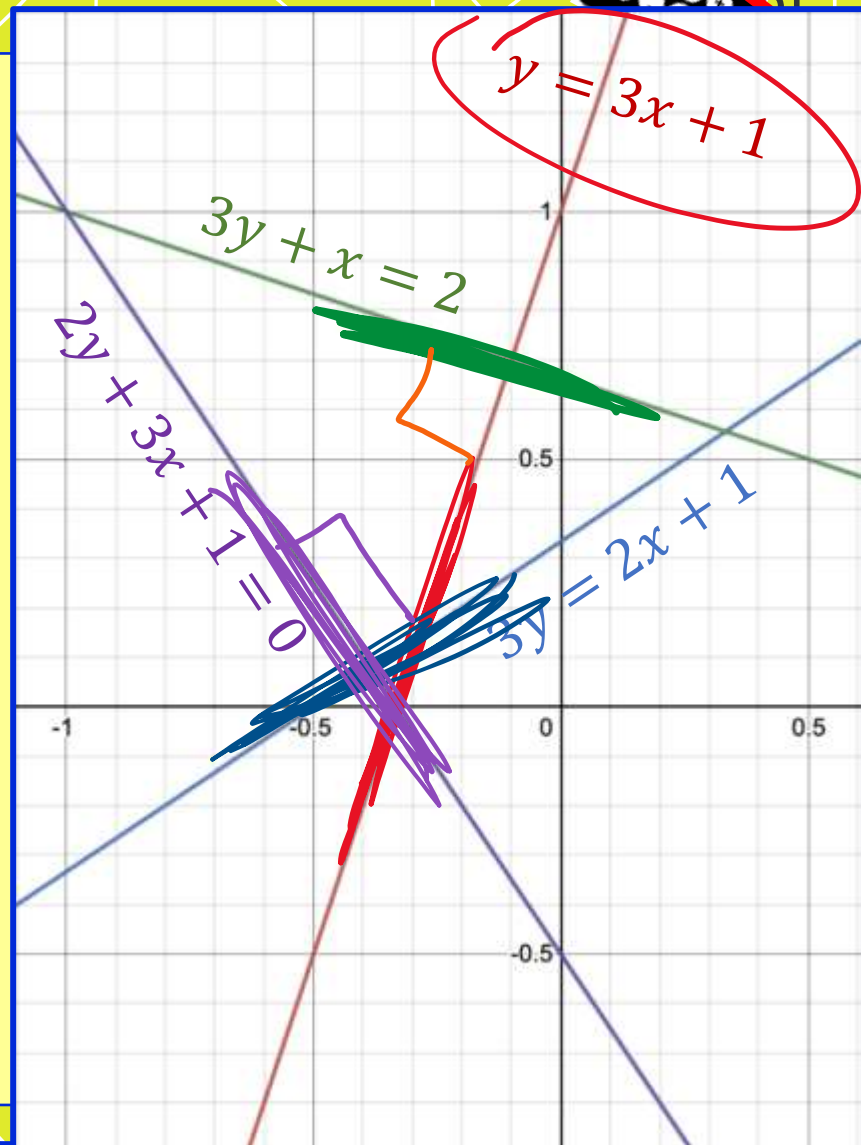
- $2y + 3x + 1 = 0$

gradient	y-intercept
$\dfrac{3}{1}$	$(0, 1)$
$\dfrac{2}{3}$	$\left(0, \dfrac{1}{3}\right)$
$-\dfrac{1}{3}$	$\left(0, \dfrac{2}{3}\right)$
$-\dfrac{3}{2}$	$\left(0, -\dfrac{1}{2}\right)$

$y = 3x + 1$

$3y + x = 2$

$2y + 3x + 1 = 0$

$3y = 2x + 1$

Linear graphs

Where do

- $y = 3x + 1$
- $3y + x = 2$

intersect?

$(-0.1, 0.7)$
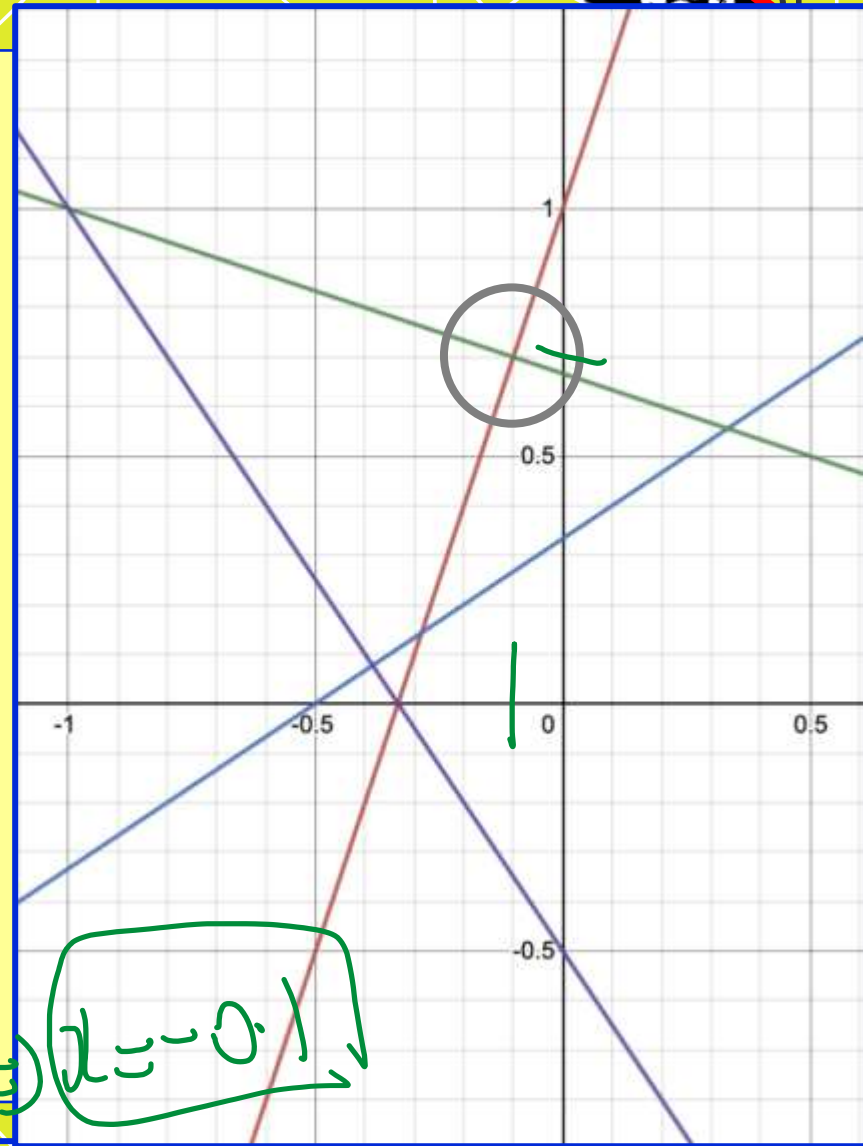
$y - 3x = 1$

$3y + x = 2 \quad \boxed{\times 3}$

$9y + 3x = 6$

$10y = 7 \Rightarrow \boxed{y = 0.7}$

$\Rightarrow 0.7 - 3x = 1$

$\Rightarrow 3x = -0.3 \Rightarrow \boxed{x = -0.1}$

Linear graphs

Where do

- $3y + x = 2$ $(-1, 1)$
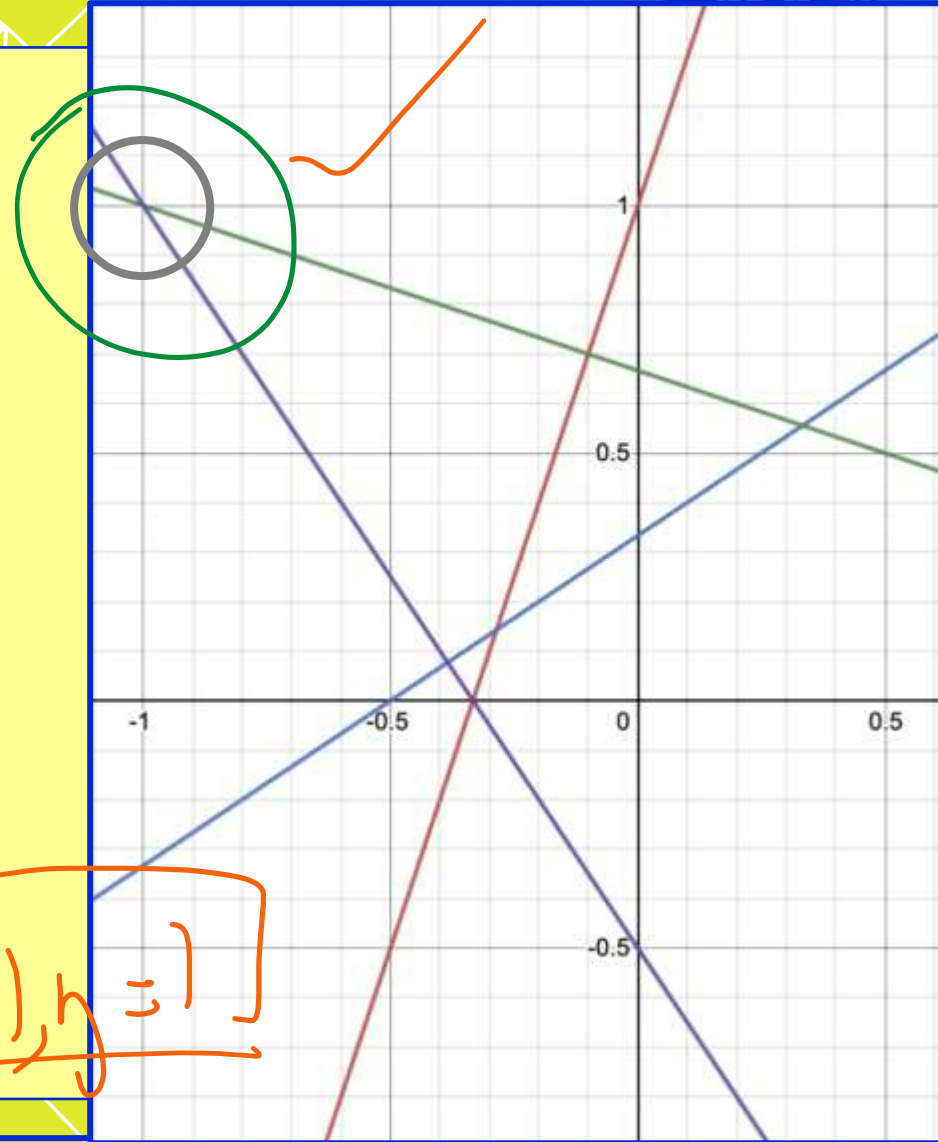- $2y + 3x + 1 = 0$

intersect?

$\Rightarrow 3y + x = 2 \quad \times 2$

$2y + 3x = -1 \quad \times 3$

$\Rightarrow 6y + 2x = 4$
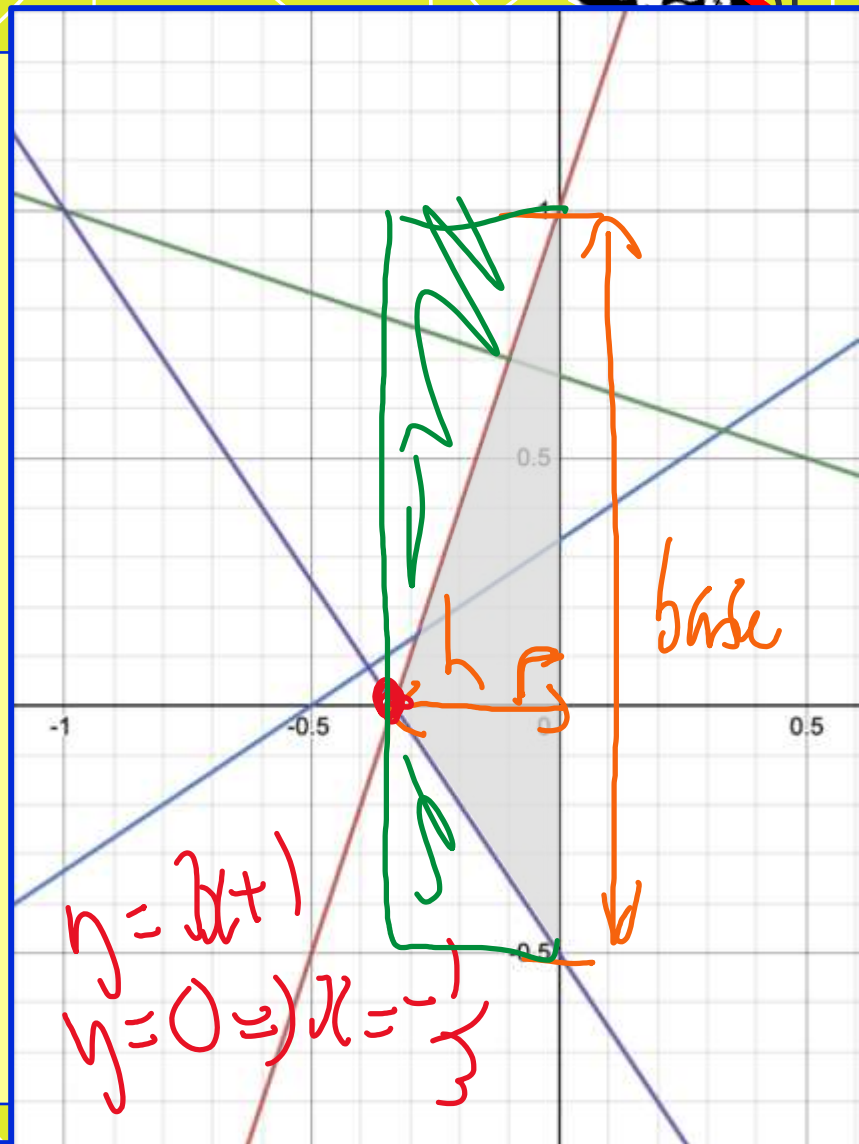
$6y + 9x = -3$

$-7x = 7 \Rightarrow x = -1, y = 1$

Linear graphs

What is the area of the shaded region?

$$base = 1 \cdot T^-$$

$$h = \frac{1}{3}$$

$$A = \frac{1}{2} \times \frac{2}{2} \times \frac{1}{3}$$

$$= \boxed{\frac{1}{4}}$$

$$y = 3x+1$$

$$y = 0 \Rightarrow x = -\frac{1}{3}$$

base

h

The equation of a straight line

Work out the equation of the straight line:

- With gradient $\frac{2}{3}$ that passes through $(0, 5)$

$$y - 5 = \frac{2}{3}(x - 0)$$

$$y = mx + c \implies y = \frac{2}{3}x + 5 \implies 3y = 2x + 15$$

- With gradient $-\frac{3}{5}$ that passes through $(-1, -4)$

$$y + 4 = \frac{-3}{5}(x + 1)$$

$$y - -4 \qquad x - -1$$

$$y = \frac{-3}{5}x + \square$$

$$\implies 5y = -3x + \square$$

$$\implies -20 = 3 + \square \implies \square = -23$$

$$\implies 5y = -3x - 23$$

$\times (-1, -4)$

The equation of a straight line

Work out the equation of the straight line:

- Parallel to $4y + 3x = 2$ that passes through $(0, -3)$

$$\Rightarrow 4y = -3x + 2$$

$$\Rightarrow m = -\frac{3}{4}$$

$m_1 = m_2$

$$\Rightarrow y = -\frac{3}{4}x + \boxed{}$$

$$\Rightarrow 4y = -3x + \boxed{}$$

$$\boxed{4y = -3x - 12}$$

$$\Rightarrow -12 = 0 + \boxed{} \Rightarrow \boxed{} = -12$$

- Perpendicular to $2y - 5x - 3 = 0$ that passes through $(1, -2)$

$$2y = 5x + 3$$

$$m = \frac{5}{2} \Rightarrow m_\perp = -\frac{2}{5}$$

$$m_1 = \frac{-1}{m_2}$$

$$\boxed{y = \frac{5}{2}x + 3}$$

The equation of a straight line

$(0, -\frac{8}{5})$

Work out the equation of the straight line:
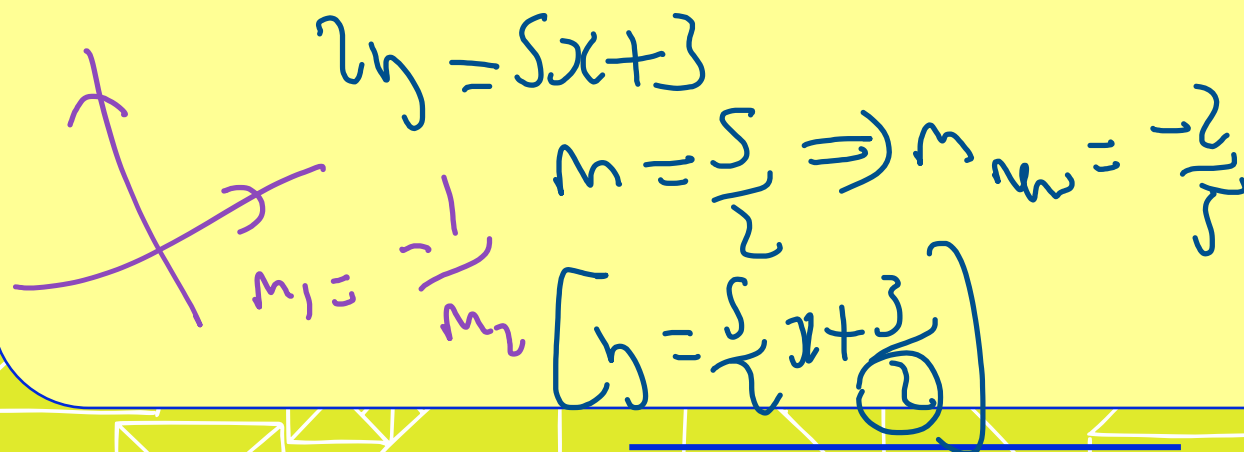
- Parallel to $4y + 3x = 2$ that passes through $(0, -3)$

$\Rightarrow y = -\frac{2}{5}x + \boxed{}$

$\Rightarrow 5y = -2x + \boxed{}$

$-10 = -2 + \boxed{}$

$\Rightarrow -8 = \boxed{} \Rightarrow 5y = -2x - 8$

- Perpendicular to $2y - 5x - 3 = 0$ that passes through $(1, -2)$

$2y = 5x + 3$

$m = \frac{5}{2} \Rightarrow m_\perp = -\frac{2}{5}$

$m_1 = \frac{-1}{m_2}$

$\boxed{y = \frac{5}{2}x + 3}$

$y = 0?$

$\Rightarrow -2x - 8 = 0$

$\Rightarrow x = -4$

$(-4, 0)$

The equation of a straight line

Work out the equation of the straight line joining:

- $(3, 2)$ and $(5, 6)$ $m = \frac{4}{2} = 2$

$(5,6)$

$(3,2)$

$\Rightarrow y = 2x + \boxed{}$

$\Rightarrow 6 = 10 + \boxed{}$

$\Rightarrow y = 2x - 4$

- $(3, 2)$ and $(5, -3)$

$(3,2)$

$(5,-3)$

$m = -\frac{5}{2} \Rightarrow y = -\frac{5}{2}x + \boxed{}$

$\Rightarrow 2y = -5x + \boxed{}$

$\ast (3,2)$

$\ast (3,-3)$

$4 = 15 + \boxed{}$

$\boxed{} = -11 \Rightarrow 2y = -5x - 11$

- $(3, 2)$ and $(3, -3)$

$x = 3$

The equation of a straight line

Work out the equation of the straight line joining:

- $(3, 2)$ and $(-1, 4)$

- **$(3, 2)$ and $(-1, 2)$**

  $(-1, 2)$ $(3, 2)$ $y = 2$
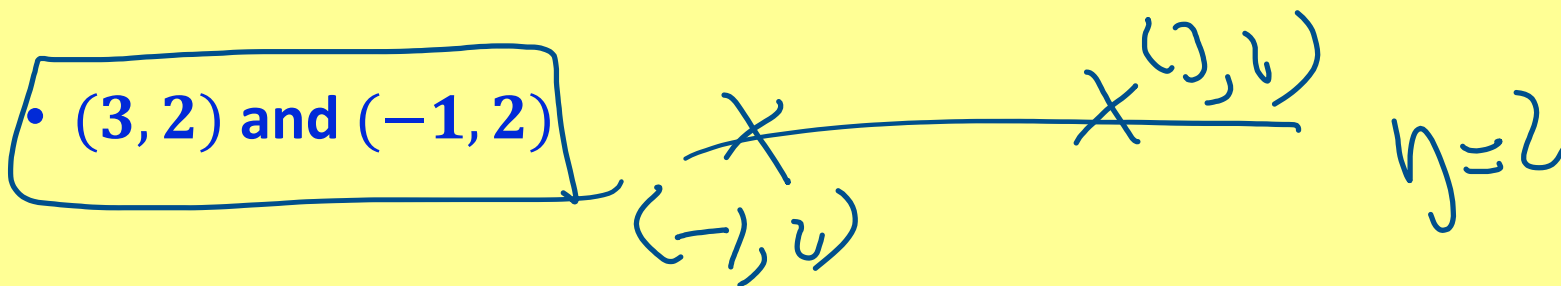
- $(3, 2)$ and $(-1, -4)$

The equation of a tangent to a circle

Work out the equation of the tangent
to this circle at the point $(-3, 6)$
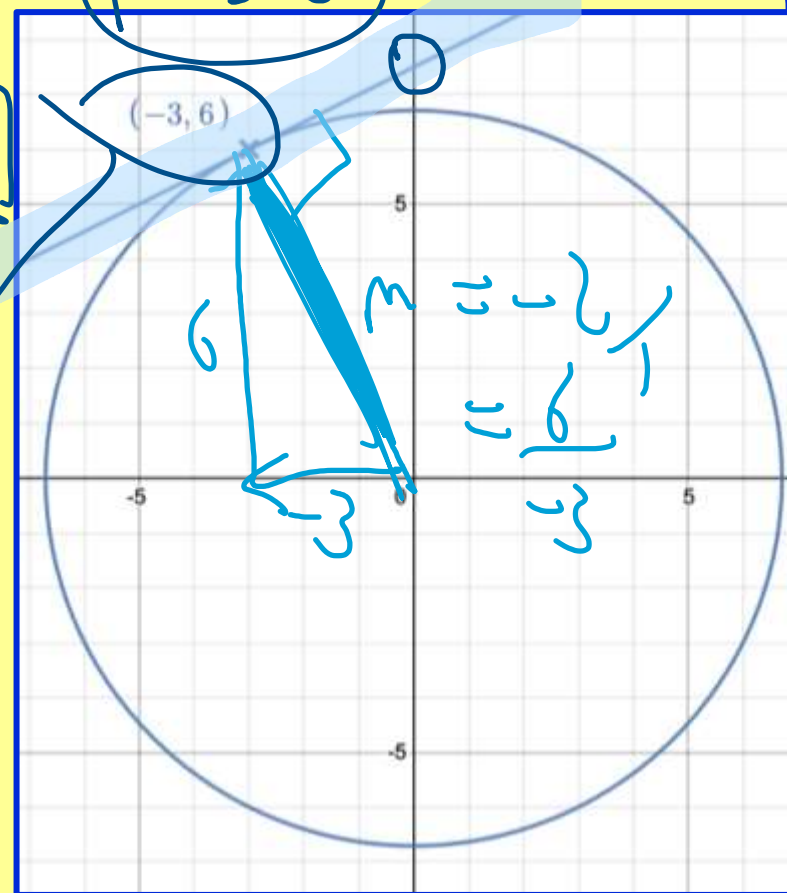
$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 45$$

$$2y = x + 15$$

$$y = \tfrac{1}{2}x + \boxed{}$$

$$\Rightarrow y = x + \boxed{}$$

$$\Rightarrow 12 = -3 + \boxed{}$$

$$\boxed{} = 15 \Rightarrow$$

$m = \tfrac{1}{2}$

$(-3, 6)$

$m = -2$

$= \dfrac{6}{-3}$

Chords and tangents

Which of these lines are chords of this circle, which are tangents to the circle, and which are neither?

- $y = 3$ chord.
- $x = 5$ tangent
- $y = 3x + 5$
- $2y = x - 12$
- $4y + 3x = 25$
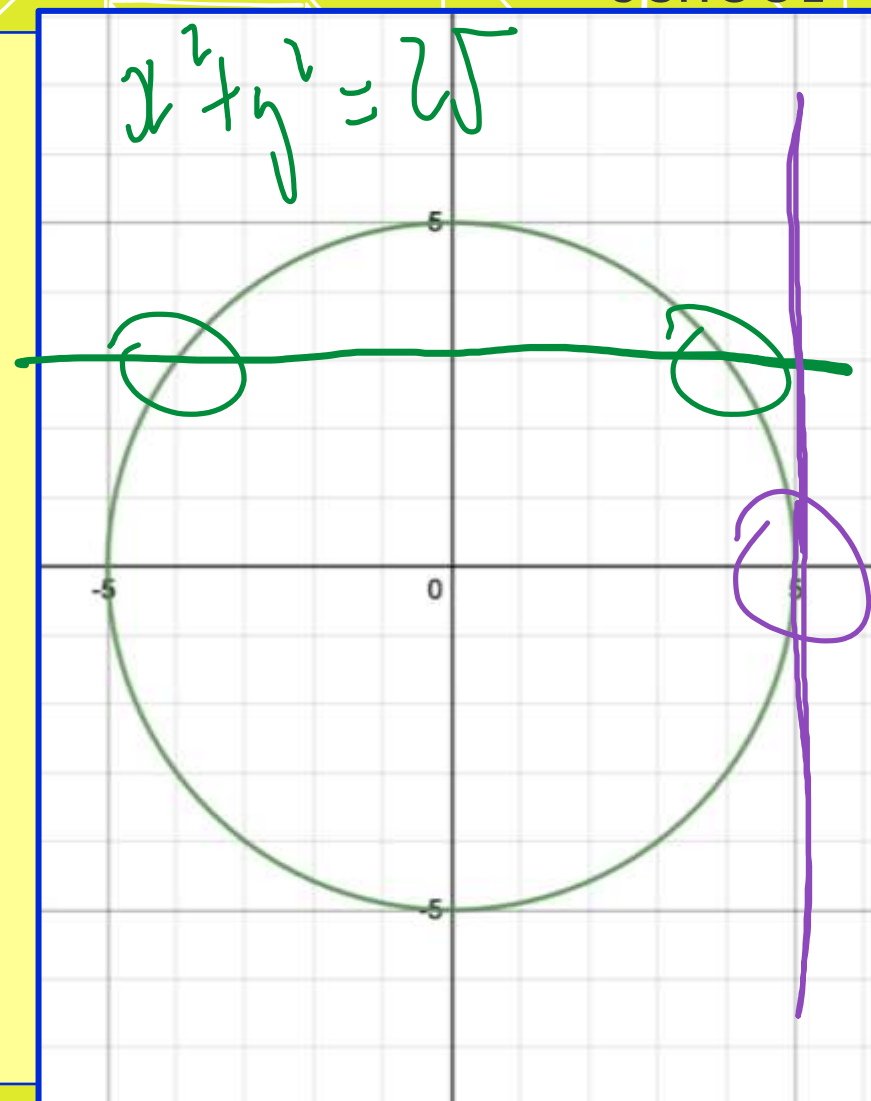
$$x^2 + y^2 = 25$$

Chrds and tangents

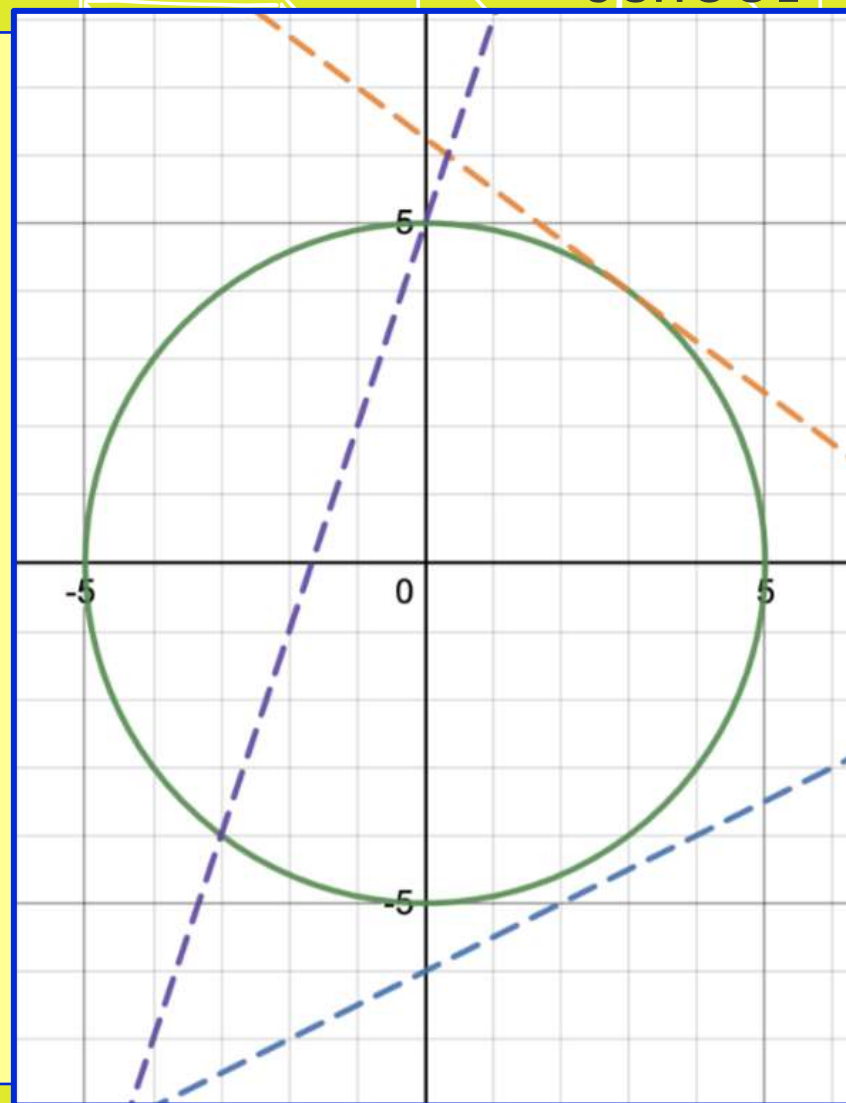Which of these lines are chords of this circle, which are tangents to the circle, and which are neither?

- $y = 3x + 5$
- $2y = x - 12$
- $4y + 3x = 25$

Chords and tangents

- $x^2 + y^2 = 25.$
- $y = 3x + 5$

$\Rightarrow x^2 + (3x + 5)^2 = 25$
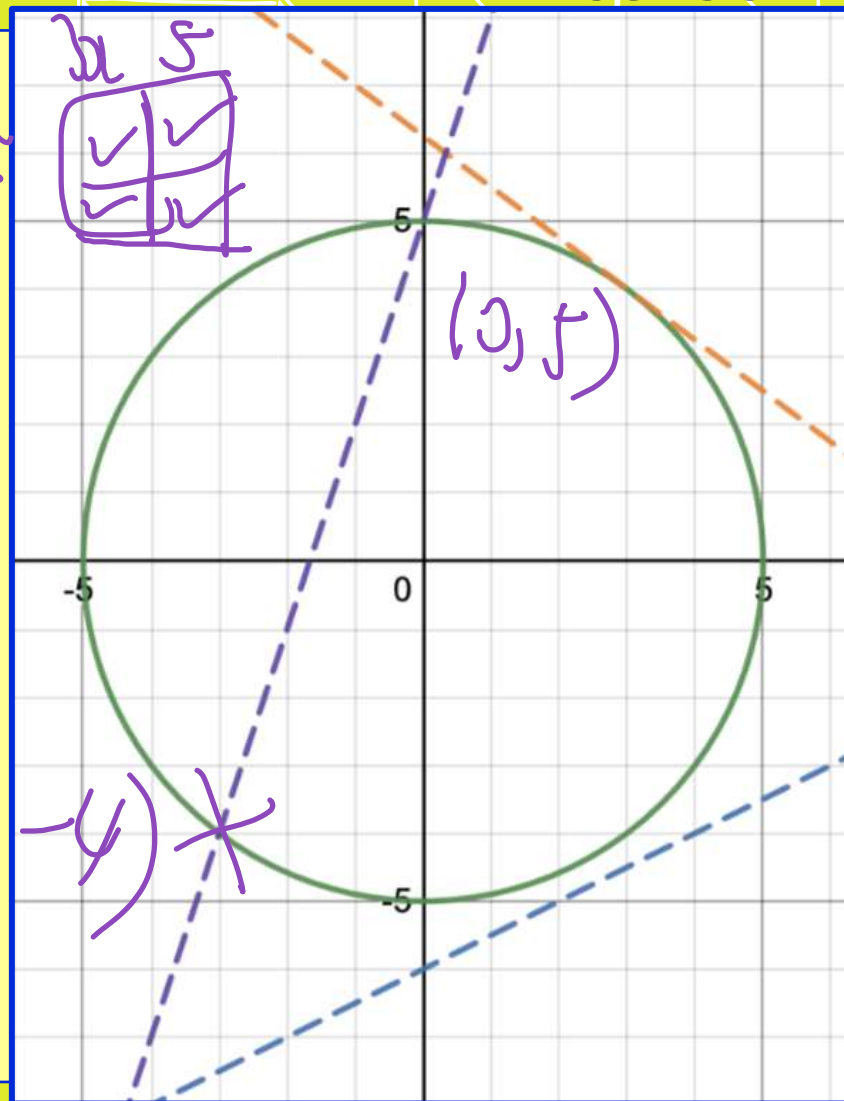
$\Rightarrow x^2 + 9x^2 + 30x + 25 = 25$

$\Rightarrow 10x^2 + 30x = 0$

$\Rightarrow x^2 + 3x = 0$

$\Rightarrow x(x + 3) = 0$

$\Rightarrow x = 0 \text{ or } x = -3$

$y = 5 \text{ or } y = -4$

$3x$
5

$(0, 5)$

$(-3, -4)$

Chords and tangents

$(-1)^2 = 4$

- $x^2 + y^2 = 25.$

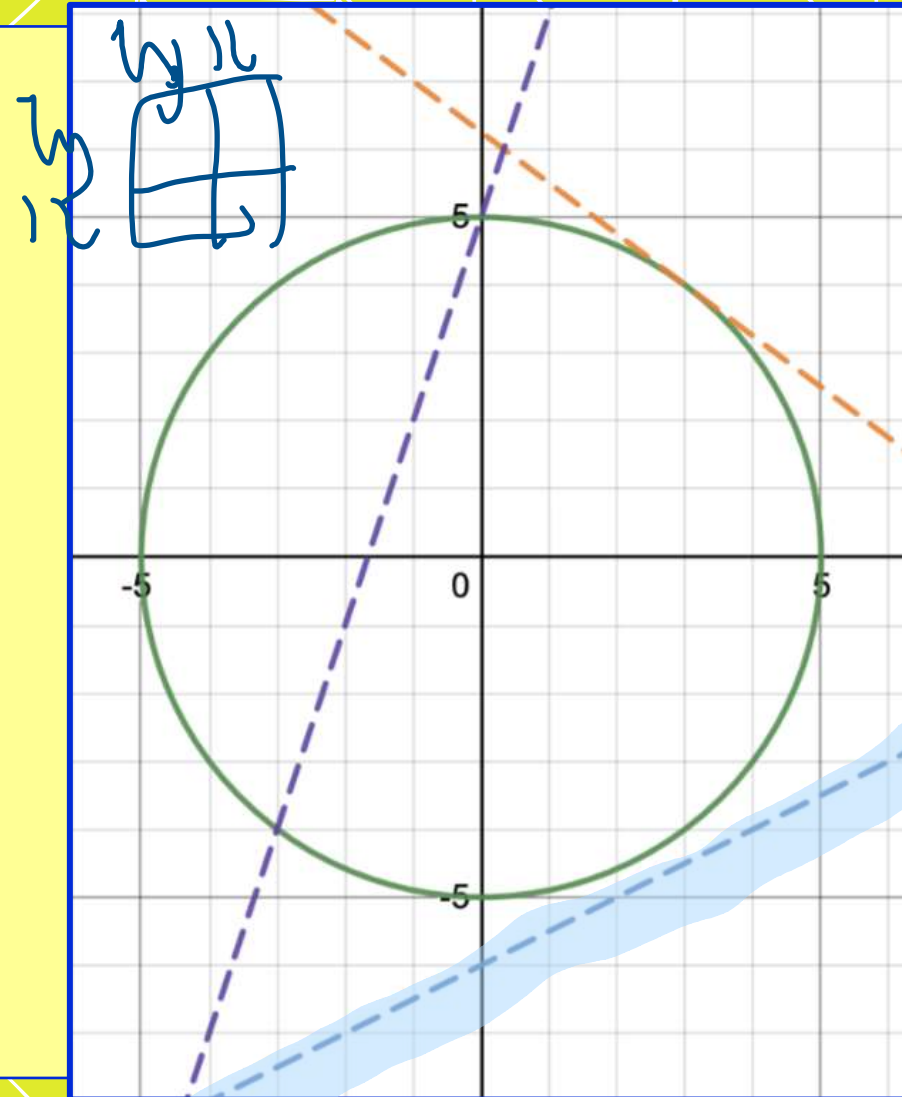- $2y = x - 12 \Rightarrow x = 2y + 12$

$\Rightarrow (2y + 12)^2 + y^2 = 25$

$\Rightarrow 4y^2 + 48y + 144 + y^2 = 25$

$\Rightarrow 5y^2 + 48y + 119 = 0$

$b^2 - 4ac = 48^2 - 4 \times 5 \times 119$

$= -76$

$y = \dfrac{-48 \pm \sqrt{-76}}{10}$

Chords and tangents

- $x^2 + y^2 = 25.$
- $4y + 3x = 25$ ← $4y + 9 = 25 - 3x$

$$\Rightarrow 4y = 25 - 3x \qquad 4b < 16$$

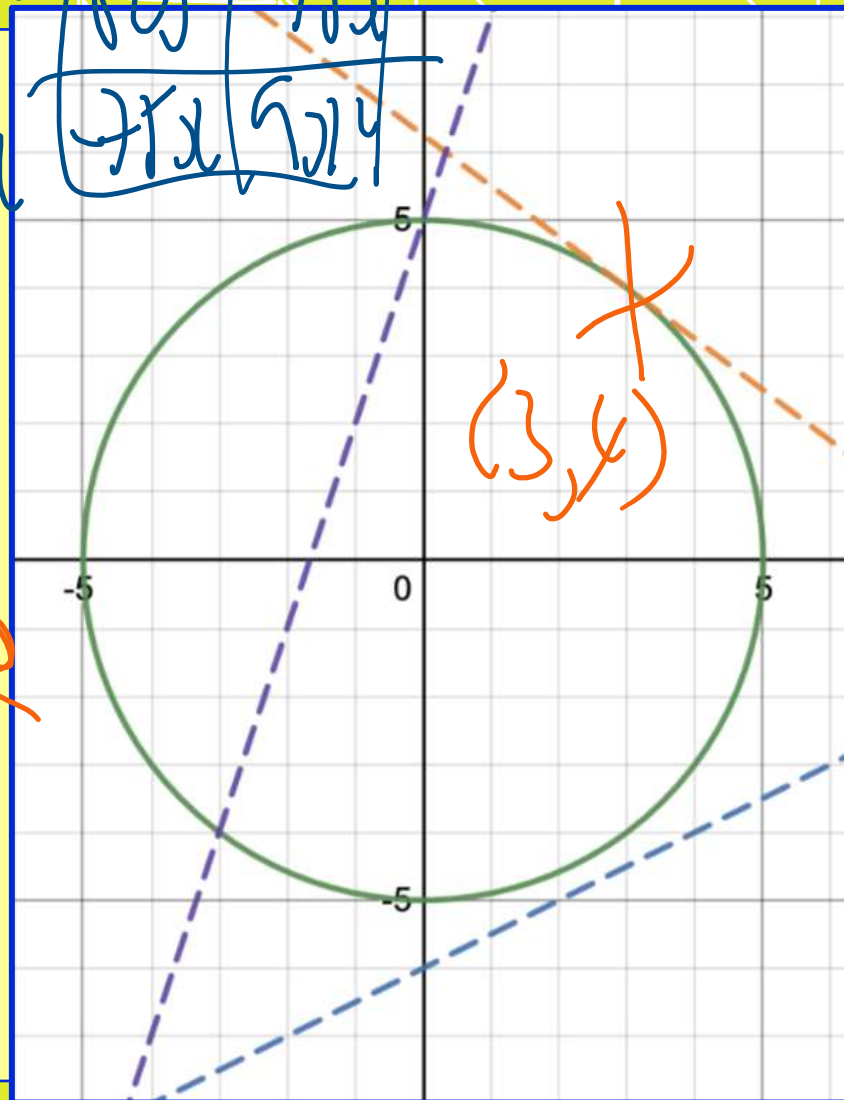$$\Rightarrow (16y)^2 = 625 - 150x + 9x^2$$

$$16x^2 + 16y^2 = 400$$

$$\Rightarrow 16x^2 + 8x^2 - 150x + 9x^2 = 400$$

$$\Rightarrow 25x^2 - 150x + 225 = 0$$

$$\Rightarrow x^2 - 6x + 9 = 0$$

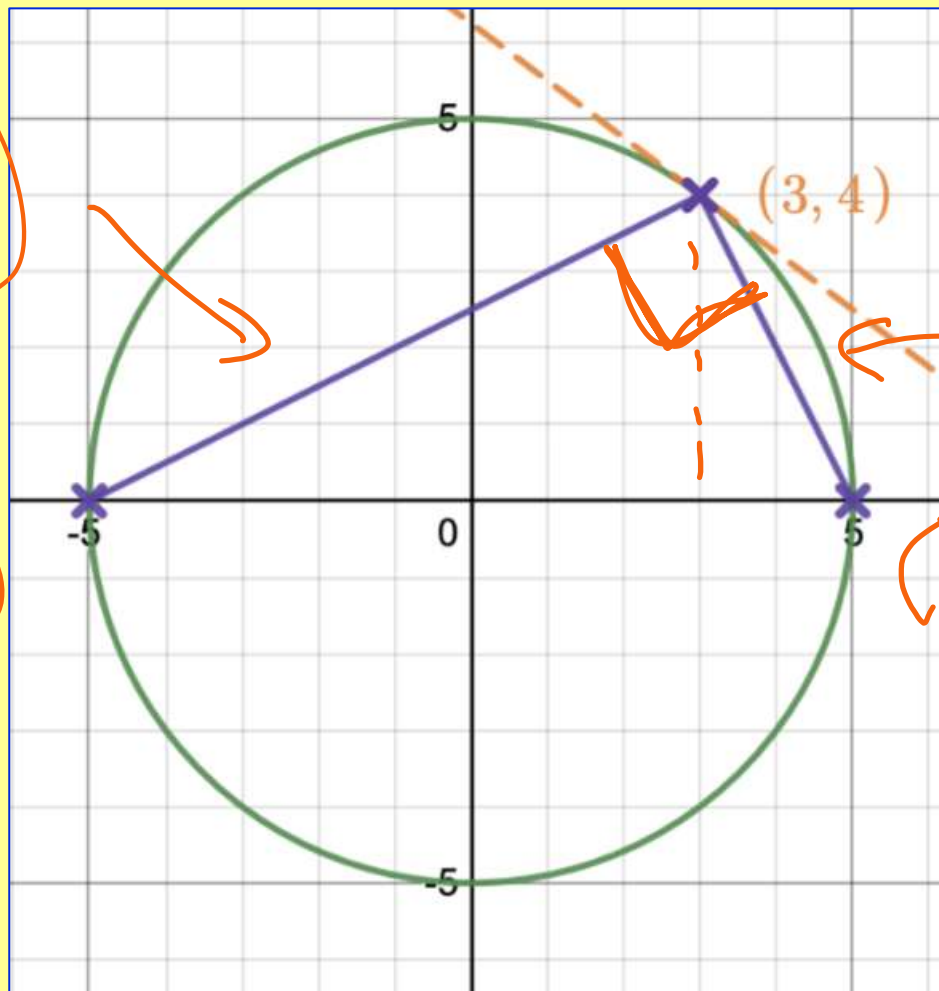$$\Rightarrow (x-3)(x-3) = 0$$

$$\Rightarrow x = 3 \quad x = 3$$

$\sqrt{(25)^2 - 7x}$

$7x \quad 5y^4$

$(3, y)$

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What do you notice?

$m = \frac{4}{2}$

$m = \frac{-2}{4}$

$(-5, 0)$

$(5, 0)$

$(3, 4)$