

**Welcome to GCSE 7+  
Friday 9 April 2021**

**Session 3:  
Fluency with quadratic expressions**

# Keep GCSE 7+ safe for everyone

- **Do not ASK** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.
- **Do not GIVE** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.
- If **anyone** asks you, in the Chat or directly, for your personal contact details, or
- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or
- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**
  - email the Head teacher [dan.abramson@kcl.ac.uk](mailto:dan.abramson@kcl.ac.uk) or text him 07902 911144 and say what your concern is,
  - or email [kclmsoutreach@kcl.ac.uk](mailto:kclmsoutreach@kcl.ac.uk) and ask Dan to contact you.

# Expanding brackets



Expand

$$\bullet (x + 3)(3x - 2) \equiv 3x^2 + 7x - 6$$

	$3x$	$-2$
$x$	$3x^2$	$5x$
$-2$	$-6x$	$-6$

$$\bullet (2x + 5)(3x - 2) \equiv 6x^2 + 11x - 10$$

	$3x$	$-2$
$2x$	$6x^2$	$11x$
$-2$	$-6x$	$-10$

$$\bullet (3x - 2)^2 \equiv$$

$$9x^2 - 12x + 4$$

	$3x$	$-2$
$3x$	$9x^2$	$-6x$
$-2$	$-6x$	$4$

$$\bullet (3x + 2)(3x - 2) \equiv 9x^2 - 4$$

	$3x$	$-2$
$3x$	$9x^2$	$6x$
$-2$	$-6x$	$-4$

# Expanding brackets



Expand

- $(2x^2 + 5)(3x^2 - 2)$

$$\equiv 6x^4 + 11x^2 - 10$$

	$3x^2$	$-2$
$2x^2$	$6x^4$	$-4x^2$
$5$	$-10x^2$	$-10$

- $(3x - 2y)^2$

$$\equiv 9x^2 - 12xy + 4y^2$$

	$3x$	$-2y$
$3x$	$9x^2$	$-6xy$
$-2y$	$-6xy$	$4y^2$

- $(3\sqrt{x} + 2)(3\sqrt{x} - 2)$

$$\equiv 9x - 4$$

	$3\sqrt{x}$	$2$
$3\sqrt{x}$	$9x$	$6\sqrt{x}$
$-2$	$-6\sqrt{x}$	$-4$

# Expanding brackets



Expand

•  $(2x + 5)(3x - 2)(x + 3)$

$$(6x^2 + 11x - 10)(x + 3)$$

$$= 6x^3 + 29x^2 + 25x - 30$$

•  $(x + 3)(3x - 2)(x - 3)$

$$= (x^2 - 9)(3x - 2)$$

$$= 3x^3 - 2x^2 - 27x + 18$$

$$\begin{array}{r} 6x^2 \quad 11x \quad -10 \\ \times \quad x \quad + \quad 3 \\ \hline 6x^3 \quad 11x^2 \quad -10x \\ 18x^2 \quad 33x \quad -30 \\ \hline 6x^3 \quad 29x^2 \quad 23x \quad -30 \end{array}$$

$$\begin{array}{r} x^2 \quad -9 \\ \times \quad 3x \quad -2 \\ \hline 3x^3 \quad -27x^2 \\ -2x^2 \quad +18 \\ \hline 3x^3 \quad -29x^2 \quad +18 \end{array}$$

# Factorising quadratic expressions



Factorise

•  $x^2 - 4x + 3$

$$\equiv (x-1)(x-3)$$

(write as a multiplication)

$$(\sim)(\sim)$$

•  $x^2 - 4x + 4$

$$\equiv (x-2)(x-2) \equiv (x-2)^2$$

•  $x^2 - 4x$

$$\equiv x(x-4)$$

•  $x^2 - 4$

$$\equiv (x-2)(x+2)$$

# Factorising quadratic expressions



Factorise

- $x^2 + 10x + 25$

$$\equiv (x+5)(x+5) \equiv (x+5)^2$$

- $9x^2 - 30x + 25$

$$\equiv (3x-5)(3x-5) \equiv (3x-5)^2$$

- $x^2 - 16$

$$\equiv (x+4)(x-4)$$

- $9x^2 - 25$   $\equiv (3x+5)(3x-5)$

# Factorising quadratic expressions



Factorise

- $3x^2 - 5x - 2$

$$\equiv (3x + 1)(x - 2)$$

	$3x$	$1$
$2$	$3x^2$	$x$
$-2$	$-6x$	$-2$

- $3x^2 + 7x - 6$

$$\equiv (3x - 2)(x + 3)$$

	$3x$	$-2$
$2$	$3x^2$	$-2x$
$3$	$9x$	$-6$

# Factorising quadratic expressions



Factorise

•  $4x^2 - 17x - 15$

$$\equiv (4x + 3)(x - 5)$$

	$x - 5$	
$4x$	$4x^2$	$-20x$
$+3$	$3x$	$-15$

•  $4x^2 + 4x - 15$

$$\equiv (2x + 5)(2x - 3)$$

	$2x + 5$	
$2x$	$4x^2$	$10x$
$-3$	$-6x$	

# Factorising quadratic? expressions



Factorise

$$\bullet x^2 - 4 \equiv (x - 2)(x + 2)$$

$$\bullet x^4 - 4x^2 \equiv x^2(x^2 - 4) \leftarrow 2/3$$
$$\equiv x^2(x - 2)(x + 2) \leftarrow 3/3$$

$$\bullet x^4 - 4 \equiv (x^2 - 2)(x^2 + 2) \quad (x^2)^2 \equiv [ ]^2$$

# Factorising quadratic? expressions



Factorise

5-  
 $x^2 - 4x + 3 \equiv (x-1)(x-3)$

Diagram illustrating the factorisation of  $x^2 - 4x + 3$ . A large purple 'U' is drawn under the expression. Green arrows point from the constant term '3' to the constant terms '-1' and '-3' in the factors. The word "y" is written in green below the arrows, indicating the constant term is split into two parts that sum to -4 and multiply to 3.

$x^4 - 4x^2 + 3 \equiv (x^2-1)(x^2-3)$   
 $(\cancel{x^2})^2 - 4(\cancel{x^2}) + 3 \equiv (x+1)(x-1)(x^2-3)$

Diagram illustrating the factorisation of  $x^4 - 4x^2 + 3$ . The expression is written in green. The first factorisation step is shown in purple. The second step shows the difference of two squares  $(x^2-1)$  being further factorised into  $(x+1)(x-1)$ . The original expression is crossed out with a green scribble.

$x^2 - 4xy + 3y^2 \equiv (x-y)(x-3y)$

Diagram illustrating the factorisation of  $x^2 - 4xy + 3y^2$ . The expression is circled in purple. To the right, a green box contains a grid for the AC method:

$x$	$-3y$
$-3y$	$+3y^2$

The grid is crossed with a green 'X'. A large green checkmark is next to it.

# Simplifying quadratic fractions



Simplify

$$\bullet \frac{x^2 - 4x}{x}$$

$$\equiv \frac{\cancel{x}(x-4)}{\cancel{x}}$$

$$\bullet \frac{x^2 - 4}{2x - 4}$$

$$\equiv \frac{(x-2)(x+2)}{2(x-2)} \equiv \frac{\cancel{x+2}}{2}$$

$$\bullet \frac{3x - 12}{x^2 - 4x}$$

$$\equiv \frac{3(x-4)}{\cancel{x}(x-4)} \equiv \frac{3}{x}$$

$$\bullet \frac{x^2 - 4}{x^2 - 4x - 12}$$

$$\equiv \frac{(x-2)(x+2)}{(x+2)(x-6)}$$

# Solving quadratic equations

Solve

•  $(x - 2)(x + 3) = 0$

$\Rightarrow x - 2 = 0$  or  $x + 3 = 0$   $x = 2$  or  $x = -3$

•  $(3x - 2)(2x + 3) = 0$

$\Rightarrow 3x - 2 = 0$  or  $2x + 3 = 0 \Rightarrow x = \frac{2}{3}$  or  $x = -\frac{3}{2}$

•  $(3x - 2)^2 = 0 \Rightarrow ( ) ( ) = 0$   
 $\Rightarrow 3x - 2 = 0 \Rightarrow x = \frac{2}{3}, \frac{2}{3}$

# Solving quadratic equations

Solve

- $x^2 - 8x - 9 = 0$

$$\Rightarrow (x - 9)(x + 1) = 0 \Rightarrow x = 9 \text{ or } x = -1$$

- $2x^2 - 3x - 9 = 0$

$$\Rightarrow (2x + 3)(x - 3) = 0 \Rightarrow x = -\frac{3}{2} \text{ or } x = 3$$

- $4x^2 - 3x = 0$

$$\Rightarrow x(4x - 3) = 0 \Rightarrow x = 0 \text{ or } x = \frac{3}{4}$$

- $4x^2 - 9 = 0 \Rightarrow (2x - 3)(2x + 3) = 0 \Rightarrow x = \frac{3}{2} \text{ or } x = -\frac{3}{2}$   
 $= \pm \frac{3}{2}$

# Solving quadratic equations

Solve

•  $(x - 2)(x + 3) = 0$  ✓  $x = 2, x = -3$

•  $(x - 2)(x + 3) = 6$

$\Rightarrow x^2 + x - 6 = 6$

$\Rightarrow x^2 + x - 12 = 0 \dots$

•  $(x - 2)(x + 3) = -6$

$\Rightarrow x^2 + x - 6 = -6$

$\Rightarrow x^2 + x = 0 \Rightarrow x(x + 1) = 0 \dots$

# Solving quadratic equations

Solve

•  $x = \frac{10}{x-3}$

$$\Rightarrow x(x-3) = 10$$

$$\Rightarrow x^2 - 3x - 10 = 0 \dots$$

# Solving quadratic equations

Solve

$$\bullet \quad 2x = \left(1 + \frac{10}{x}\right)x$$

$$\Rightarrow 2x^2 = x + \cancel{\frac{10}{x}x}$$

$$\Rightarrow 2x^2 = x + 10$$

$$\Rightarrow 2x^2 - x - 10 = 0 \dots$$

# Solving quadratic equations

Solve

$$\frac{2x^2}{1} \left( \frac{5}{x} - \frac{2}{x^2} \right) = 2x^2$$

$$\Rightarrow \frac{5x^2}{x} - \frac{2x^2}{x^2} = 2x^2$$

$$\Rightarrow 5x - 2 = 2x^2$$

$$\Rightarrow \text{—————} = 0$$

# Solving quadratic equations

Solve

$$\frac{6}{x-1} + \frac{4}{x} = 3x(x-1)$$

$$\Rightarrow \frac{6x(x-1)}{x-1} + \frac{4x(x-1)}{x} = 3x^2 - 3x$$

$$\Rightarrow 6x + 4x - 4 = 3x^2 - 3x$$

$$\Rightarrow \underline{\hspace{10em}} = 0$$

# Solving quadratic equations

Solve

•  $36x^2 + 33x - 20 = 0$

$a = 36$   $b = +33$  ,  $c = -20$

$\frac{5}{12}$  ,  $-\frac{4}{3}$

$$x = \frac{-33 \pm \sqrt{(33)^2 - 4 \times 36 \times (-20)}}{72}$$

*Discriminant*

$$= \frac{-33 \pm \sqrt{3869}}{72} = \frac{-33 \pm 63}{72} = \frac{30}{72} , \frac{-96}{72}$$

# Solving quadratic equations

Solve

- $x^2 + 4x - 2 = 0$

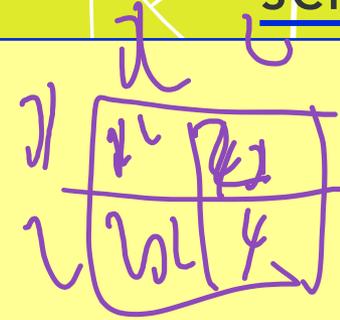
$$\begin{aligned} \Rightarrow x &= \frac{-4 \pm \sqrt{6 - 4 \times 1 \times -2}}{2} \\ &= \frac{-4 \pm \sqrt{14}}{2} = \frac{-4 \pm \sqrt{2} \sqrt{7}}{2} \\ &= -2 \pm \sqrt{7} \end{aligned}$$

# Completing the square

Complete the square in

•  $x^2 + 4x + 4$  ✓

$$\equiv (x+2)^2 + 0$$



•  $x^2 + 4x + 6$

$$\equiv (x+2)^2 + 2$$

•  $x^2 + 4x + 1$   $\equiv (x+2)^2 - 3 \equiv \cancel{(x+1)^2 + 6}$

# Completing the square

Complete the square in

•  $x^2 - 6x + 2$

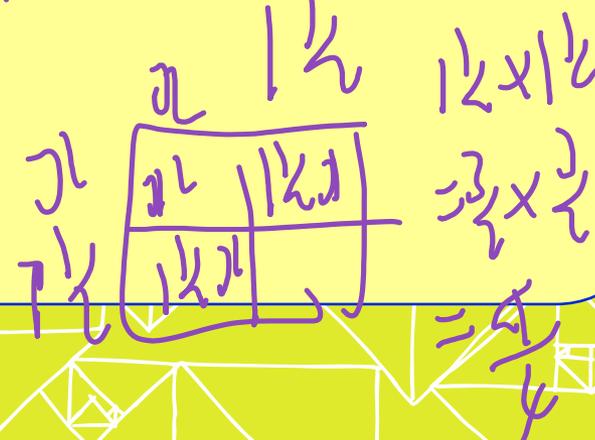
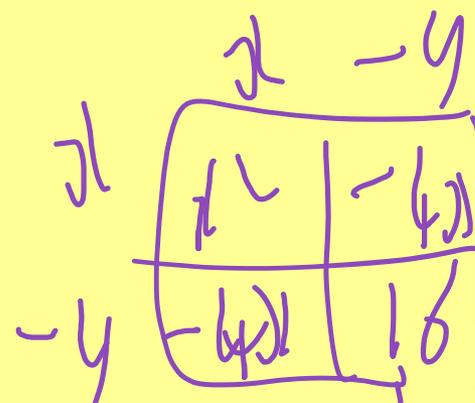
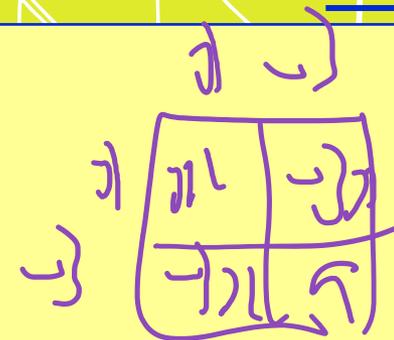
$$\equiv (x - 3)^2 - 7$$

•  $x^2 - 8x + 11$

$$\equiv (x - 4)^2 - 5$$

•  $x^2 + 3x + 1$

$$\equiv \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$$



# Solving quadratic equations ...

... by completing the square

- $x^2 - 4x + 1 = 0$

$$(x-2)^2 - 3 = 0$$

$$\Rightarrow (x-2)^2 = 3 \Rightarrow x-2 = \pm\sqrt{3}$$

- $x^2 - 4x + 1 = 2x - 5$

$$\Rightarrow x = 2 \pm \sqrt{3}$$

$$\Rightarrow x^2 - 6x + 6 = 0$$

$$\Rightarrow (x-3)^2 - 3 = 0$$

$$\Rightarrow x-3 = \pm\sqrt{3}$$

$$\Rightarrow x = 3 \pm \sqrt{3}$$

$$\begin{array}{|c|c|} \hline x & -6 \\ \hline -2x & 4 \\ \hline \end{array}$$

# GCSE 7+: what now?

- 0930-1030: all together, for revision teaching
- 1045-1145: you work independently on the practice questions we will give you
- 1145-1245: in break-out rooms, in small groups with two teachers, you can go over the “practice permanent” questions, ask your own questions, and demonstrate your **knowledge, fluency and understanding**

Return here in time  
for **starting** the  
examples classes at  
1145. Don't be late!