

Welcome to GCSE 7+

Friday 9 April 2021

Session 3:

Fluency with quadratic expressions

Keep GCSE 7+ safe for everyone

- **Do not ASK** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.
- **Do not GIVE** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.
- If **anyone** asks you, in the Chat or directly, for your personal contact details, or
- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or
- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**
 - email the Head teacher dan.abramson@kcl.ac.uk or text him 07902 911144 and say what your concern is,
 - or email kclmsoutreach@kcl.ac.uk and ask Dan to contact you.

Expanding brackets



Expand

$$\bullet (x + 3)(3x - 2) \equiv 3x^2 + 7x - 6$$

	$3x$	-2
x	$3x^2$	$-2x$
3	$-6x$	-6

$$\bullet (2x + 5)(3x - 2) \equiv 6x^2 + 11x - 10$$

	$3x$	-2
$2x$	$6x^2$	$-4x$
5	$-10x$	-10

$$\bullet (3x - 2)^2 \equiv$$

$$9x^2 - 12x + 4$$

	$3x$	-2
$3x$	$9x^2$	$-6x$
-2	$-6x$	4

$$\bullet (3x + 2)(3x - 2) \equiv 9x^2 - 4$$

	$3x$	-2
$3x$	$9x^2$	$-6x$
2	$-6x$	-4

Expanding brackets



Expand

- $(2x^2 + 5)(3x^2 - 2)$

$$\equiv 6x^4 + 11x^2 - 10$$

	$3x^2$	-2
$2x^2$	$6x^4$	$-4x^2$
5	$-10x^2$	-10

- $(3x - 2y)^2$

$$\equiv 9x^2 - 12xy + 4y^2$$

	$3x$	$-2y$
$3x$	$9x^2$	$-6xy$
$-2y$	$-6xy$	$4y^2$

- $(3\sqrt{x} + 2)(3\sqrt{x} - 2)$

$$\equiv 9x - 4$$

	$3\sqrt{x}$	2
$3\sqrt{x}$	$9x$	$6\sqrt{x}$
-2	$-6\sqrt{x}$	-4

Expanding brackets



Expand

- $(2x + 5)(3x - 2)(x + 3)$

$$(6x^2 + 11x - 10)(x + 3)$$

$$= 6x^3 + 25x^2 + 23x - 30$$

- $(x + 3)(3x - 2)(x - 3)$

$$= (x^2 - 9)(3x - 2)$$

$$= 3x^3 - 2x^2 - 27x + 18$$

$$\begin{array}{r} 6x^2 \quad 11x \quad -10 \\ \times \quad x + 3 \\ \hline 6x^3 \quad 11x^2 \quad -10x \\ 18x^2 \quad 33x \quad -30 \\ \hline 6x^3 \quad 25x^2 \quad 23x \quad -30 \end{array}$$

$$\begin{array}{r} x^2 \quad -9 \\ \times \quad 3x - 2 \\ \hline 3x^3 \quad -27x^2 \\ -2x^2 \quad +18 \\ \hline 3x^3 \quad -29x^2 \quad +18 \end{array}$$

Factorising quadratic expressions



Factorise

- $x^2 - 4x + 3$

$$\equiv (x-1)(x-3)$$

(Cointe as a multiplication

$$(\sim)(\sim)$$

- $x^2 - 4x + 4$

$$\equiv (x-2)(x-2) \equiv (x-2)^2$$

- $x^2 - 4x$

$$\equiv x(x-4)$$

- $x^2 - 4$

$$\equiv (x-2)(x+2)$$

Factorising quadratic expressions



Factorise

- $x^2 + 10x + 25$

$$\equiv (x+5)(x+5) \equiv (x+5)^2$$

- $9x^2 - 30x + 25$

$$\equiv (3x-5)(3x-5) \equiv (3x-5)^2$$

- $x^2 - 16$

$$\equiv (x+4)(x-4)$$

- $9x^2 - 25$ $\equiv (3x+5)(3x-5)$

Factorising quadratic expressions



Factorise

- $3x^2 - 5x - 2$

$$\equiv (3x + 1)(x - 2)$$

	$3x$	1
2	$3x^2$	x
-2	$-6x$	-2

- $3x^2 + 7x - 6$

$$\equiv (3x - 2)(x + 3)$$

	$3x$	-2
2	$3x^2$	$-2x$
3	$9x$	-6

Factorising quadratic expressions



Factorise

- $4x^2 - 17x - 15$

$$\equiv (4x + 3)(x - 5)$$

	x	-5
$4x$	$4x^2$	$-20x$
$+3$	$3x$	-15

- $4x^2 + 4x - 15$

$$\equiv (2x + 5)(2x - 3)$$

	$2x$	$+5$
$2x$	$4x^2$	$10x$
-3	$-6x$	

Factorising quadratic? expressions



Factorise

• $x^2 - 4 \equiv (x-2)(x+2)$

• $x^4 - 4x^2 \equiv x^2(x^2 - 4) \leftarrow 2/2$
 $\equiv x^2(x-2)(x+2) \leftarrow 3/3$

• $x^4 - 4 \equiv (x^2-2)(x^2+2) \quad (x^4)^2 \equiv []^2$

Factorising quadratic? expressions



Factorise

• $x^2 - 4x + 3 \equiv (x-1)(x-3)$

Diagram showing the factorisation of $x^2 - 4x + 3$. A large purple 'U' is drawn under the expression. Green arrows point from the constant term '3' to the constant terms '-1' and '-3' in the factors. The word "y" is written in green below the arrows, indicating the constant term is split into two parts that multiply to give the constant term.

• $x^4 - 4x^2 + 3 \equiv (x^2-1)(x^2-3)$

Diagram showing the factorisation of $x^4 - 4x^2 + 3$. The expression is written as $(x^2)^2 - 4(x^2) + 3$. The factors are $(x^2-1)(x^2-3)$. The first factor is further factorised into $(x+1)(x-1)$, resulting in the final factorisation: $(x+1)(x-1)(x^2-3)$.

• $x^2 - 4xy + 3y^2 \equiv (x-y)(x-3y)$

Diagram showing the factorisation of $x^2 - 4xy + 3y^2$. The expression is circled in purple. The factors are $(x-y)(x-3y)$. To the right, a grid-in method is shown for finding the factors:

x	$-y$
x	$-3y$
$-3y$	$+3y^2$

The grid-in method shows the correct factors $(x-y)(x-3y)$ and the resulting terms x^2 , $-3y^2$, $-3xy$, and $+3y^2$. A large green checkmark is next to the grid.

Simplifying quadratic fractions



Simplify

$$\bullet \frac{x^2 - 4x}{x}$$

$$= \frac{\cancel{x}(x-4)}{\cancel{x}}$$

$$\bullet \frac{x^2 - 4}{2x - 4}$$

$$= \frac{(x-2)(x+2)}{2(x-2)} = \frac{\cancel{x+2}}{2}$$

$$\bullet \frac{3x - 12}{x^2 - 4x}$$

$$= \frac{3(x-4)}{x(x-4)} = \frac{3}{x}$$

$$\bullet \frac{x^2 - 4}{x^2 - 4x - 12}$$

$$= \frac{(x-2)(x+2)}{(x+2)(x-6)} = \frac{\cancel{x+2}}{\cancel{x+2}(x-6)} = \frac{1}{x-6}$$

Solving quadratic equations

Solve

- $(x - 2)(x + 3) = 0$

$$\Rightarrow x - 2 = 0 \quad \text{or} \quad x + 3 = 0 \quad x = 2 \quad \text{or} \quad x = -3$$

- $(3x - 2)(2x + 3) = 0$

$$\Rightarrow 3x - 2 = 0 \quad \text{or} \quad 2x + 3 = 0 \Rightarrow x = \frac{2}{3} \quad \text{or} \quad x = -\frac{3}{2}$$

- $(3x - 2)^2 = 0 \Rightarrow () () = 0$
 $\Rightarrow 3x - 2 = 0 \Rightarrow x = \frac{2}{3}, \frac{2}{3}$

Solving quadratic equations

Solve

- $x^2 - 8x - 9 = 0$

$$\Rightarrow (x - 9)(x + 1) = 0 \Rightarrow x = 9 \text{ or } x = -1$$

- $2x^2 - 3x - 9 = 0$

$$\Rightarrow (2x + 3)(x - 3) = 0 \Rightarrow x = -\frac{3}{2} \text{ or } x = 3$$

- $4x^2 - 3x = 0$

$$\Rightarrow x(4x - 3) = 0 \Rightarrow x = 0 \text{ or } x = \frac{3}{4}$$

- $4x^2 - 9 = 0 \Rightarrow (2x - 3)(2x + 3) = 0 \Rightarrow x = \frac{3}{2}, -\frac{3}{2}$
 $= \pm \frac{3}{2}$

Solving quadratic equations

Solve

- $(x - 2)(x + 3) = 0$ ✓ $x = 2, x = -3$

- $(x - 2)(x + 3) = 6$
 $\Rightarrow x^2 + x - 6 = 6$
 $\Rightarrow x^2 + x - 12 = 0 \dots$

- $(x - 2)(x + 3) = -6$
 $\Rightarrow x^2 + x - 6 = -6$
 $\Rightarrow x^2 + x = 0 \Rightarrow x(x + 1) = 0 \dots$

Solving quadratic equations

Solve

• $x = \frac{10}{x-3}$

$$\Rightarrow x(x-3) = 10$$

$$\Rightarrow x^2 - 3x - 10 = 0 \dots$$

Solving quadratic equations

Solve

$$\bullet \quad 2x = \left(1 + \frac{10}{x}\right)x$$

$$\Rightarrow 2x^2 = x + \cancel{\frac{10}{x}x}$$

$$\Rightarrow 2x^2 = x + 10$$

$$\Rightarrow 2x^2 - x - 10 = 0 \dots$$

Solving quadratic equations

Solve

$$\frac{5}{x} - \frac{2}{x^2} = 2x$$

$$\Rightarrow \frac{5x}{x^2} - \frac{2}{x^2} = 2x$$

$$\Rightarrow \frac{5x - 2}{x^2} = 2x$$

$$\Rightarrow \frac{5x - 2}{x^2} = 0$$

Solving quadratic equations

Solve

$$\frac{6}{x-1} + \frac{4}{x} = 3x(x-1)$$

$$\Rightarrow \frac{6x(x-1)}{x-1} + \frac{4x(x-1)}{x} = 3x^2 - 3x$$

$$\Rightarrow 6x + 4x - 4 = 3x^2 - 3x$$

$$\Rightarrow \quad = 0$$

Solving quadratic equations

Solve

• $36x^2 + 33x - 20 = 0$

$a = 36$ $b = +33$, $c = -20$

$\frac{5}{12}$, $-\frac{4}{3}$

$x = \frac{-33 \pm \sqrt{(33)^2 - 4 \times 36 \times (-20)}}{72}$

$= \frac{-33 \pm \sqrt{3869}}{72} = \frac{-33 \pm 63}{72} = \frac{30}{72}$, $-\frac{96}{72}$

Discriminant

Solving quadratic equations

Solve

- $x^2 + 4x - 2 = 0$

$$\begin{aligned} \Rightarrow x &= \frac{-4 \pm \sqrt{6 - 4 \times 1 \times -2}}{2} \\ &= \frac{-4 \pm \sqrt{10}}{2} = \frac{-4 \pm \sqrt{2} \sqrt{5}}{2} \\ &= -2 \pm \sqrt{5} \end{aligned}$$

Completing the square

Complete the square in

- $x^2 + 4x + 4$ ✓

$$\equiv (x+2)^2 + 0$$

Handwritten diagram illustrating the completion of the square for $x^2 + 4x + 4$. The diagram shows a 2x2 grid with x^2 in the top-left, $4x$ in the top-right, and 4 in the bottom-right. Above the grid, x is written above the first column and 2 above the second column. To the left of the grid, x is written to the left of the first row and 2 to the left of the second row. Arrows point from the x and 2 labels to the corresponding terms in the grid.

- $x^2 + 4x + 6$

$$\equiv (x+2)^2 + 2$$

- $x^2 + 4x + 1$ $\equiv (x+2)^2 - 3 \equiv \cancel{(x+1)^2 + 6}$

Completing the square

Complete the square in

• $x^2 - 6x + 2$

$$\equiv (x - 3)^2 - 7$$

• $x^2 - 8x + 11$

$$\equiv (x - 4)^2 - 5$$

• $x^2 + 3x + 1$

$$\equiv \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$$

$$\begin{array}{c} x \quad -3 \\ \begin{array}{|c|c|} \hline x^2 & -6x \\ \hline -6x & 9 \\ \hline \end{array} \\ -3 \end{array}$$

$$\begin{array}{c} x \quad -4 \\ \begin{array}{|c|c|} \hline x^2 & -8x \\ \hline -8x & 16 \\ \hline \end{array} \\ -4 \end{array}$$

$$\begin{array}{c} x \quad \frac{3}{2} \\ \begin{array}{|c|c|} \hline x^2 & 3x \\ \hline 3x & \frac{9}{4} \\ \hline \end{array} \\ \frac{3}{2} \end{array} \quad \begin{array}{l} \frac{1}{2} \times \frac{1}{2} \\ = \frac{1}{4} \times \frac{1}{4} \\ = \frac{1}{4} \end{array}$$

Solving quadratic equations ...

... by completing the square

- $x^2 - 4x + 1 = 0$

$$(x-2)^2 - 3 = 0$$

$$\Rightarrow (x-2)^2 = 3 \Rightarrow x-2 = \pm\sqrt{3}$$

- $x^2 - 4x + 1 = 2x - 5$

$$\Rightarrow x = 2 \pm \sqrt{3}$$

$$\Rightarrow x^2 - 6x + 6 = 0$$

$$\Rightarrow (x-3)^2 - 3 = 0$$

$$\Rightarrow x-3 = \pm\sqrt{3}$$

$$\Rightarrow x = 3 \pm \sqrt{3}$$

$$\begin{array}{r} x-2 \\ \hline x^2-4x+1 \\ \hline -2x+4 \\ \hline 4 \end{array}$$

GCSE 7+: what now?

- 0930-1030: all together, for revision teaching
- 1045-1145: you work independently on the practice questions we will give you
- 1145-1245: in break-out rooms, in small groups with two teachers, you can go over the “practice questions” questions, ask your own questions, and check your **knowledge, fluency and understanding**

Return here in time
for **starting** the
examples classes at
1145. Don't be late!