

Welcome to GCSE 7+

Thursday 8 April 2021

Session 2:

Algebra with indices and surds

Keep GCSE 7+ safe for everyone

- **Do not ASK** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.
- **Do not GIVE** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.
- If **anyone** asks you, in the Chat or directly, for your personal contact details, or
- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or
- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**
 - email the Head teacher dan.abramson@kcl.ac.uk or text him 07902 911144 and say what your concern is,
 - or email kclmsoutreach@kcl.ac.uk and ask Dan to contact you.

Simplifying expressions with indices



Simplify

$$\bullet \sqrt{x^8} = (x^8)^{\frac{1}{2}} = x^4$$

$$\bullet \sqrt[3]{y^{12}} = (y^{12})^{\frac{1}{3}} = y^4$$

$$\bullet \sqrt{36x^{36}} = (\underline{36} x^{36})^{\frac{1}{2}} = 6 x^{18}$$

$$\bullet \sqrt[3]{216y^{216}} = 6 (y^{216})^{\frac{1}{3}} = 6 y^{72}$$

$$\bullet \sqrt{25x^{16}} + 3(x^2)^4 = 5x^8 + 3x^8 = 8x^8$$

Simplifying expressions with indices



Simplify

$$\begin{aligned}
 & \bullet 12x^5y^1 \div 4x^2y^3 = \frac{12}{4} x^{5-2} y^{1-3} = 3x^3y^{-2} = \frac{3x^3}{y^2} \left(\frac{3x^3 \times \frac{1}{y^2}}{1} \right) \\
 & \bullet \sqrt{25x^{16}} \div 3(x^2)^4 = 5x^8 \div 3x^8 = \frac{5x^8}{3x^8} = \frac{5}{3} \\
 & \bullet (3x^2y^3)^3 \times (3x^3y^2)^{-2} = \frac{27x^6y^9}{9x^6y^4} = 3y^5 \\
 & \bullet (4x^6y^{-8})^{-\frac{1}{2}} = \frac{1}{\sqrt{4x^6y^{-8}}} = \frac{1}{2x^3y^{-4}} = \frac{1}{2x^3} \times \frac{y^4}{1} = \frac{y^4}{2x^3}
 \end{aligned}$$

Simplifying expressions with indices



Simplify

• $x^{-3} = \frac{1}{x^3}$

• $2x^{-3} = \frac{2}{x^3}$

• $2x^{-3} \times (3x)^{-2} = \frac{2}{x^3} \times \frac{1}{(3x)^2} = \frac{2}{x^3 \times 9x^2} = \frac{2}{9x^5}$

• $3x^{-2} \div (2x)^{-3} = \frac{3}{x^2} \div \frac{1}{(2x)^3} = \frac{3}{x^2} \times \frac{(2x)^3}{1} = \frac{3 \times 8x^3}{x^2} = \frac{24x^3}{x^2} = 24x$

Simplifying expressions with indices



Simplify

$$\bullet 2x^{-3} + 3x^{-3} = \frac{2}{x^3} + \frac{3}{x^3} = \frac{5}{x^3}$$

$$\bullet 2x^{-3} + 3x^{-2} = \frac{2 \times 1}{x^3 \times 1} + \frac{3 \times 1}{x^2 \times x} = \frac{2}{x^3} + \frac{3x}{x^3} = \frac{2+3x}{x^3}$$

(Note: The original image contains a circled '2' and 'CD' next to the first term, which are likely annotations or corrections.)

$$\bullet (2x)^{-3} + (3x)^{-2} = \frac{1 \times 8x}{8x^3 \times 1} + \frac{1 \times 9x}{9x^2 \times 1} = \frac{8}{8x^3} + \frac{9x}{9x^2} = \frac{8+9x}{8x^3}$$

Simplifying expressions with indices



Simplify

$$\bullet \frac{6x^2}{9x} = \frac{2x^2}{3}$$

$$\bullet \frac{6x^2y}{9xy^3} = \frac{2x}{3y^2}$$

$$\bullet \frac{6+2x}{3x+x^2} =$$

$$\frac{2(3+x)}{x(3+x)} = \frac{2}{x} \quad \checkmark$$

$$\bullet \frac{4x^2+6x}{10x^4+15x^3} =$$

$$\frac{2x(2x+3)}{5x^3(2x+3)} = \frac{2x}{5x^2} = \frac{2}{5x}$$

$$\frac{1(2x)}{5x^3}$$

Solving equations with indices



Solve

• $x^3 = 3\frac{3}{8}$

$$x = \sqrt[3]{3\frac{3}{8}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2} = 1\frac{1}{2}$$

• $x^{-2} = 2\frac{1}{4}$

$$= \frac{1}{x^2} = \frac{9}{4} \Rightarrow x^2 = \frac{4}{9} \Rightarrow x = \pm \frac{2}{3}$$

• $x^{\frac{2}{3}} = 6.25$

$$\Rightarrow \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(6.25\right)^{\frac{3}{2}} \Rightarrow x = \frac{125}{8}$$

"6.25"

• $x^{-\frac{3}{2}} = 0.064$

$=$

Solving equations with indices



Solve

- $x^3 = 2^{15}$

$$\Rightarrow x = \sqrt[3]{2^{15}} = (2^{15})^{\frac{1}{3}} = 2^5 = 32$$

- $x^{-2} = 4^7$

$$\frac{1}{x^2} = 4^7 \Rightarrow x^2 = \frac{1}{4^7} \Rightarrow x = \pm \sqrt{\frac{1}{4^7}} = \pm \frac{1}{2^{14}}$$

- $2^x = 4^{12}$

$$(2^2)^x = 2^{12}$$

$$\Rightarrow 2x = 12$$

$$\Rightarrow x = 6$$

- $4^x = 2^{12}$

$$(2^2)^x = 2^{12} \Rightarrow 2^{2x} = 2^{12} \Rightarrow 2x = 12 \Rightarrow x = 6$$

- $4^x = 8^{-4}$

Solving equations with surds



Solve

$$\begin{aligned} \bullet \sqrt{3}x + 3 &= 6 \Rightarrow \sqrt{3}x = 3 \Rightarrow x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{3\sqrt{3}}{\sqrt{3}} = \sqrt{3} \end{aligned}$$
$$\begin{aligned} \bullet \sqrt{3}x + 3 &= \boxed{\sqrt{27}x} + 6 \\ \Rightarrow \sqrt{3}x + 3 &= 3\sqrt{3}x + 6 \\ \Rightarrow -3 &= 2\sqrt{3}x \Rightarrow x = \frac{-3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}}{2 \times 3} = \frac{-\sqrt{3}}{2} \end{aligned}$$

Solving equations with surds



Solve

- $\sqrt{3}x + 3 = 2x + 6$

$$\Rightarrow -3 = 2x - \sqrt{3}x$$

$$\Rightarrow -3 = x(2 - \sqrt{3}) \Rightarrow x = \frac{-3}{2 - \sqrt{3}}$$

"x)"

$$\frac{-3}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

conjugate surds

- $\sqrt{3}x + 3 = \sqrt{2}x + 6$

$$\Rightarrow -3 = \sqrt{2}x - \sqrt{3}x$$

$$= x(\sqrt{2} - \sqrt{3}) \Rightarrow x = \frac{-3}{\sqrt{2} - \sqrt{3}} \times \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \dots$$

Make x the subject of ...

- $ax + b = c$

$$\Rightarrow ax = c - b \Rightarrow x = \frac{c - b}{a}$$

- $a\sqrt{x} + b = c$

$$\Rightarrow a\sqrt{x} = c - b \Rightarrow \sqrt{x} = \frac{c - b}{a} \Rightarrow x = \left(\frac{c - b}{a}\right)^2$$

- $a\sqrt{x + b} = c$

$$\Rightarrow \sqrt{x + b} = \frac{c}{a} \Rightarrow x + b = \left(\frac{c}{a}\right)^2 \Rightarrow x = \left(\frac{c}{a}\right)^2 - b$$

Make x the subject of ...

- $ax + bx = c$

$$\Rightarrow x(a+b) = c \Rightarrow x = \frac{c}{a+b}$$

- $x\sqrt{a} = b$

$$\Rightarrow x = \frac{b}{\sqrt{a}}$$

- $x\sqrt{a} + bx = c$

$$\Rightarrow x(\sqrt{a} + b) = c \Rightarrow x = \frac{c}{\sqrt{a} + b}$$

Make x the subject of ...

• $\frac{a}{x} + b = c$

$\Rightarrow \frac{a}{x} = c - b \Rightarrow a = x(c - b)$

$\Rightarrow \frac{a}{c - b} = x$

• $\frac{a - bx}{x} = c$

$\Rightarrow a - bx = cx$

$\Rightarrow a = cx + bx \Rightarrow a = x(c + b)$

$\Rightarrow x = \frac{a}{c + b}$

• $\frac{\sqrt{x}}{a} = \frac{\sqrt{x} - b}{c}$

$\Rightarrow c\sqrt{x} = a(\sqrt{x} - b) \Rightarrow c\sqrt{x} = a\sqrt{x} - ab$
 $\Rightarrow c\sqrt{x} - a\sqrt{x} = -ab$

Make x the subject of ...

- $\frac{a}{x} + b = c$

$$\rightarrow \sqrt{x}(c-b) = -ab$$

$$\Rightarrow \sqrt{x} = \frac{-ab}{c-b} \Rightarrow x = \left(\frac{-ab}{c-b} \right)^2$$

- $\frac{a-bx}{x} = c$

- ~~$\frac{\sqrt{x}}{a} = \frac{\sqrt{x}-b}{c}$~~ ~~$\times$~~ ~~$\cancel{ac}$~~ ~~$\times$~~ ~~$\cancel{ac}$~~

$$\Rightarrow c\sqrt{x} = a(\sqrt{x}-b) \Rightarrow c\sqrt{x} = a\sqrt{x} - ab$$

$$\Rightarrow c\sqrt{x} - a\sqrt{x} = -ab$$

Solving equations with surds



Solve

$$\sqrt{2}x + 3y = 1$$

$$x - \sqrt{2}y = 3\sqrt{2}$$

$\times \sqrt{2}$

$\times 3$

$$\begin{aligned} \Rightarrow 2x + 3\sqrt{2}y &= \sqrt{2} \\ + 3x - 3\sqrt{2}y &= 6\sqrt{2} \end{aligned}$$

$$\hline 5x = 7\sqrt{2}$$

$$\Rightarrow \boxed{x = \frac{7\sqrt{2}}{5}}$$

$$\sqrt{2} \left(\frac{7\sqrt{2}}{5} \right) + 3y = 1$$

$$\Rightarrow \frac{7}{5} + 3y = 1$$

$$\Rightarrow 3y = -\frac{2}{5}$$

$$\Rightarrow y = -\frac{2}{15}$$

Solving equations with surds



Solve

$$y = \sqrt{5}x - 8$$

$$\sqrt{5}y - 2x = \sqrt{5}$$

$$\Rightarrow \sqrt{5}(\sqrt{5}x - 8) - 2x = \sqrt{5}$$

$$\Rightarrow (\sqrt{5}x) - 8\sqrt{5} - 2x = \sqrt{5}$$

$$\Rightarrow 3x = 9\sqrt{5}$$

$$\Rightarrow \boxed{x = 3\sqrt{5}}$$

$$\Rightarrow \sqrt{5}(3\sqrt{5}) - 8 = 7$$