

Welcome to GCSE 7+

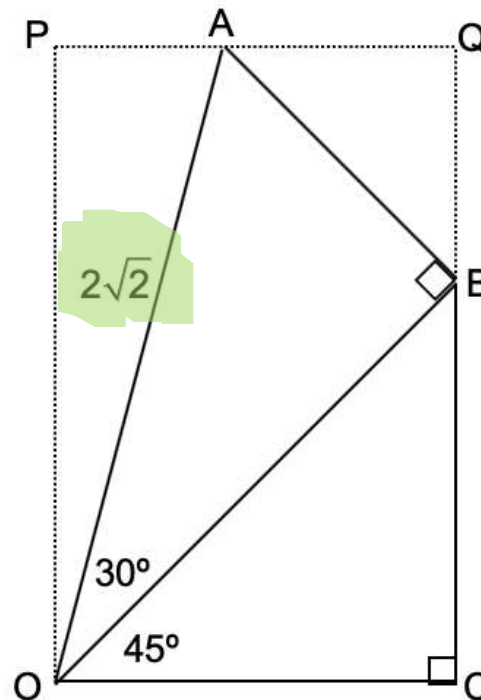
Tuesday 13 April 2021

Session 7:

Ratio, percentages, proportion

A puzzle to ponder

In the diagram, OCQP is a rectangle, $OA = 2\sqrt{2}$, angle $AOB = 30^\circ$ and angle $BOC = 45^\circ$. Use this diagram to find exact values for the sine, cosine and tangent ratios for 15° and 75° .



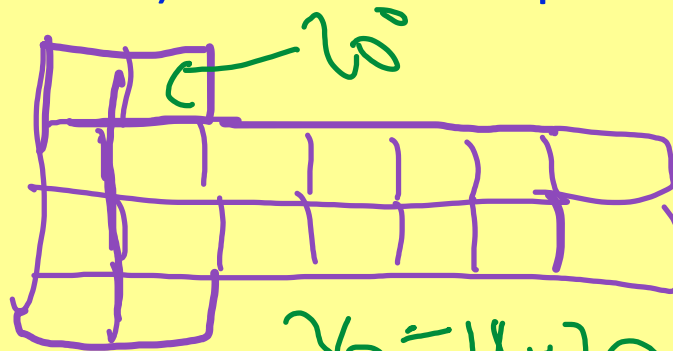
Keep GCSE 7+ safe for everyone

- **Do not ASK** anyone for **their** personal contact details: email, 'phone number, social media name, Instagram address etc.
- **Do not GIVE** anyone **your** personal contact details: email, 'phone number, social media name, Instagram address etc.
- If **anyone** asks you, in the Chat or directly, for your personal contact details, or
- If you read in the Chat, or if you overhear, **anyone** asking for or giving out personal contact details, or
- If you have any concerns about the welfare/wellbeing of any participant, including yourself, then you must **as soon as possible**
 - email the Head teacher dan.abramson@kcl.ac.uk or text him **07902 911144** and say what your concern is,
 - or email kclmsoutreach@kcl.ac.uk and ask Dan to contact you.

Quadrilaterals



- a) The interior angles of a quadrilateral are in the ratio 2 : 7 : 7 : 2 (in order). What is the quadrilateral?

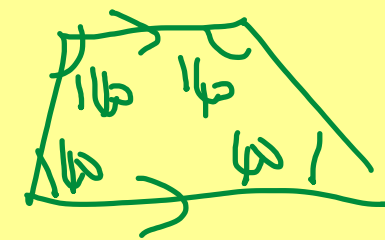


$$360 \div 18 = 20$$

$$= 360^\circ$$

$$\begin{array}{c} 60^\circ \\ 140^\circ \\ 140^\circ \\ 60^\circ \end{array}$$

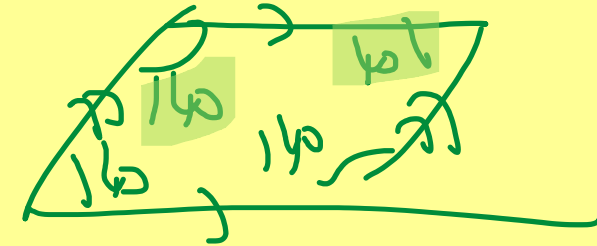
$$360^\circ$$



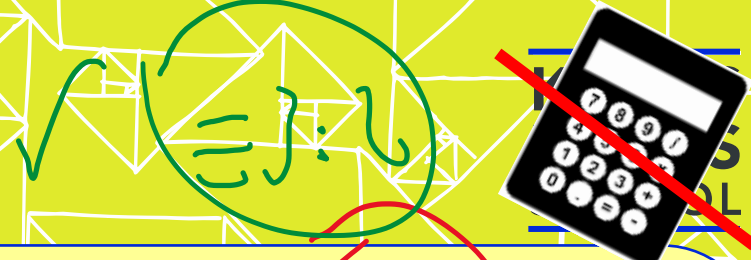
- b) The interior angles of a quadrilateral are in the ratio 2 : 7 : 2 : 7 (in order). What is the quadrilateral?

$$40^\circ, 140^\circ, 40^\circ, 140^\circ$$

$$360^\circ$$



One ratio tells us a lot!



- In a box of Lego™, the ratio of blue to red pieces is 6 : 4
- Therefore ...



50% more B than R

33 1/3% less R than B

$$\frac{6}{4} = 1.5 = \frac{3}{2} \times \frac{2}{3}$$

$$R = \frac{4}{10} = \frac{2}{5} \text{ of total}$$

$$B = \frac{6}{10} = \frac{3}{5} \text{ of total}$$

One ratio tells us a lot!



- In a box of Lego™, the ratio of blue to red pieces is 6 : 4
- Therefore ...



$$b:r = 6:4$$

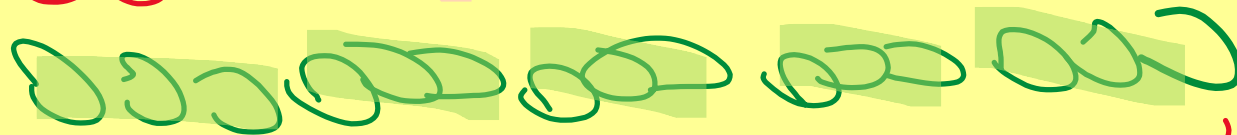
$$\frac{b}{6} = \frac{r}{4}$$



One ratio tells us a lot!



- In a box of Lego™, the ratio of blue to red to green pieces is 6 : 9 : 15 $\Rightarrow 2 : 3 : 5$
- Therefore ...



$$6 = \frac{15}{30} = \frac{1}{5} \text{ of total}$$

And 60% free

Interpreting ratios



- A rectangle has sides in the ratio 3 : 5. The area is 735 cm². What is the perimeter?

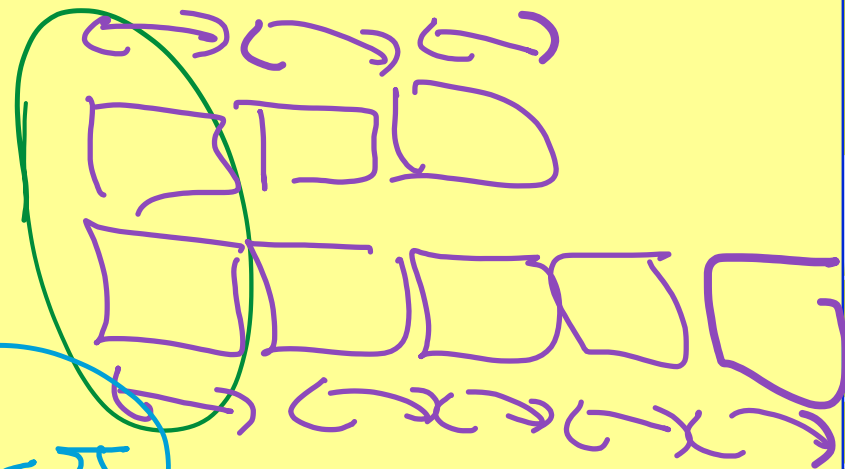
$$W:L = 3:5$$

$$\Rightarrow \frac{W}{3} = \frac{L}{5}$$

$$\Rightarrow W = \frac{3L}{5}$$

$$\frac{3L}{5} \times L = 735$$

$$W = 3 \times \frac{75}{5} = 21$$



$$L = 75$$

$$\frac{3L}{5} \times L = 735$$

$$\Rightarrow \frac{3L}{5} = 735$$

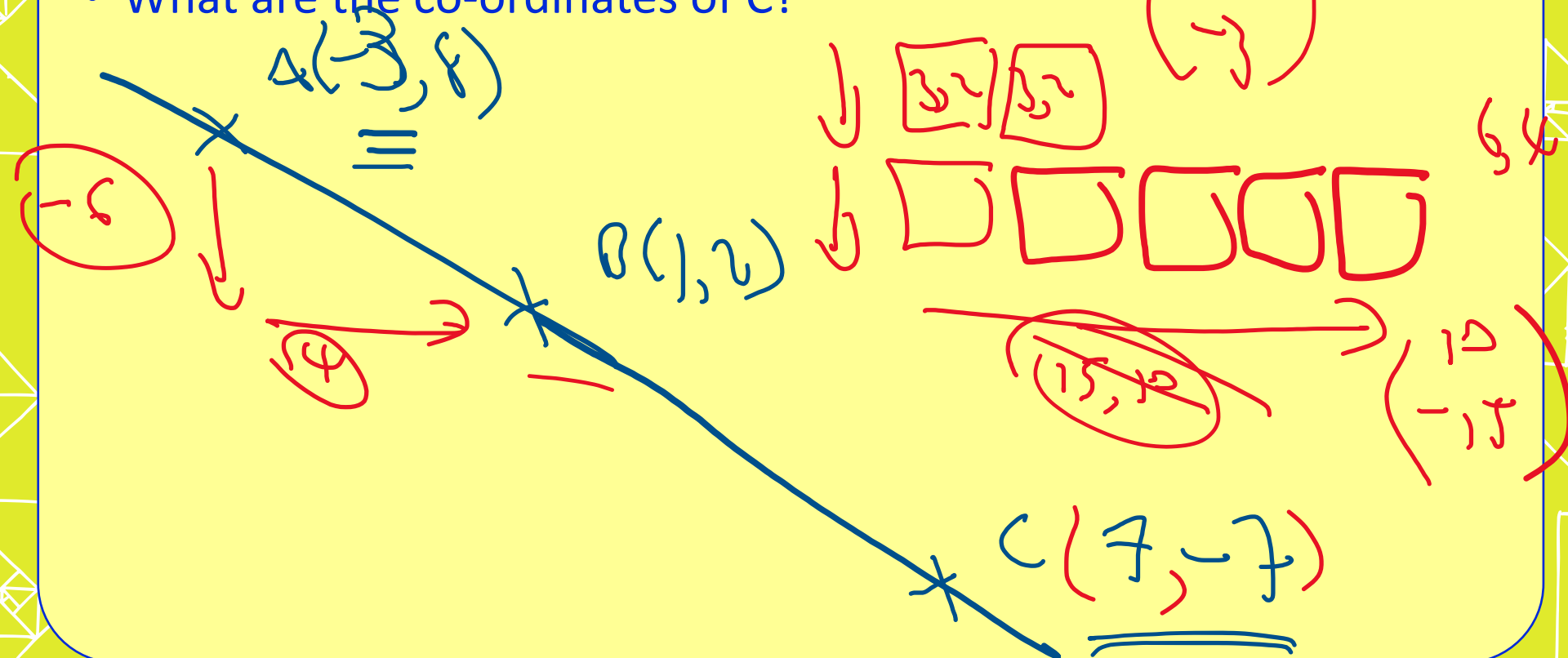
$$L = 75$$

$$\Rightarrow L^2 = 1725$$

Interpreting ratios



- A has co-ordinates $(-3, 8)$ and B has co-ordinates $(1, 2)$.
- C is on the line AB, and $AB : BC = 2 : 5$
- What are the co-ordinates of C?



Interpreting ratios



- The ratio of the sum to the difference of two numbers is 5 : 2.
What is the ratio of the numbers?

x, y

$$x + y : x - y = 5 : 2$$

$$\Rightarrow \frac{x+y}{x-y} = \frac{5}{2}$$



$$\Rightarrow 2x + 2y = 5x - 5y$$

$$\Rightarrow \frac{2y}{3x} = \frac{3}{7} \Rightarrow y : x = 3 : 7$$

Changing ratios

$$a:b = 7:3 \Rightarrow \frac{a}{7} = \frac{b}{3}$$



- A and B share some money in the ratio 7 : 3. *A gets a, B gets b*
- A gives B £10. The ratio of their shares is now 3 : 2.
- How much do they each have?

$$a - 10 : b + 10 = 3 : 2$$

$$\Rightarrow \frac{a - 10}{3} = \frac{b + 10}{2}$$

"A"

"B"

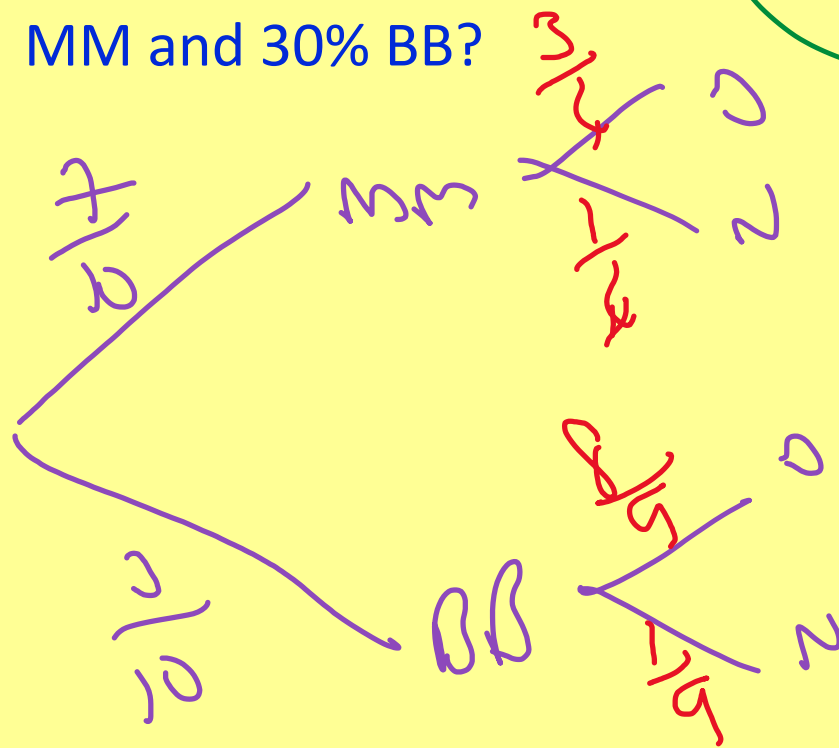
$$\Rightarrow 2a - 20 = 3b + 30$$

$$\Rightarrow 2a - 3b = 50 \quad \& \quad 3a = 7b \quad \dots$$

(£70 → 60
£30 → 40)

Combining ratios

- *Morning Munch* contains oats : nuts in the ratio 3 : 1
- *Breakfast Boost* contains oats : nuts in the ratio 8 : 1
- What is the overall ratio of oats : nuts in a mixture that is 70% MM and 30% BB?



$$= \frac{7}{10} \times \frac{3}{4} = \frac{21}{40}$$

$$= \frac{3}{10} \times \frac{8}{5} = \frac{24}{50}$$

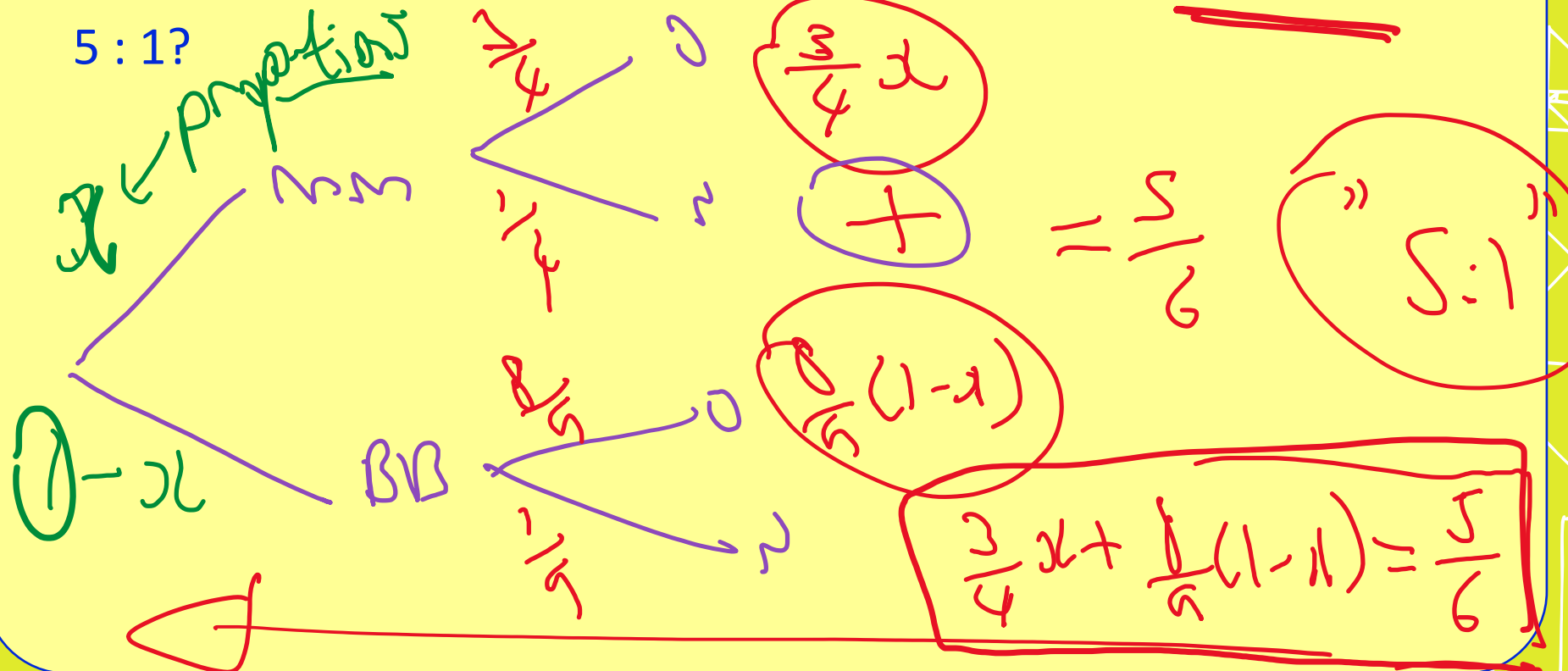
$$\begin{array}{r} 189 \\ 360 \\ \hline 285 \\ 360 \end{array}$$

$$\begin{array}{r} 56 \\ 360 \end{array}$$

$$285 : 75 = \dots$$

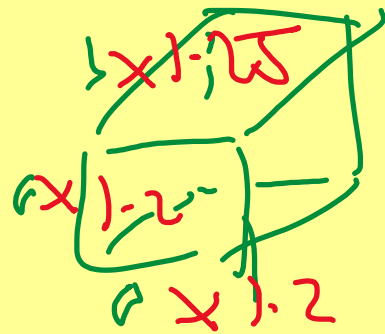
Combining ratios

- *Morning Munch* contains oats : nuts in the ratio 3 : 1
- *Breakfast Boost* contains oats : nuts in the ratio 8 : 1
- What mixture of MM and BB has oats : nuts in the overall ratio 5 : 1?



Percentage changes ...

- A cuboid has dimensions a cm by a cm by b cm.
- When a increases ^{$\times 1.2$} by 20% and b increases by 25%, what is the percentage increase in the volume? ^{$\times 1.25$}



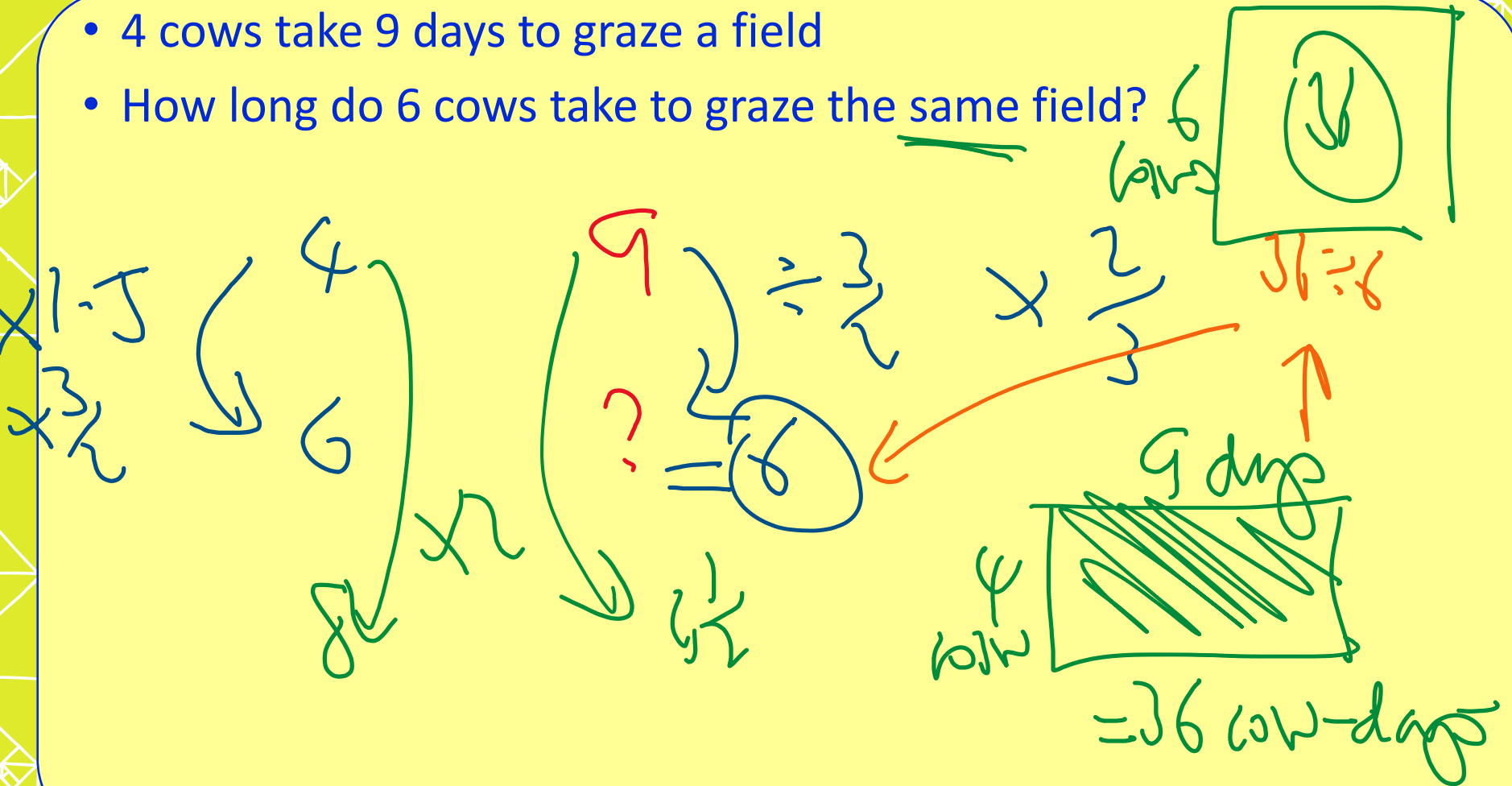
$$V = a^2 b$$

$$\begin{aligned} V &= 1.2a \times 1.2a \times 1.25b \\ &= 1.44 \times 1.25 b \end{aligned}$$

$$\text{80\% inc} \Rightarrow \underline{\underline{1.8a^2 b}}$$

Proportional relationships

- 4 cows take 9 days to graze a field
- How long do 6 cows take to graze the same field?



Proportional relationships

- 4 cows take 9 days to graze 1000m².
- How long do 6 cows take to graze 2500m²?

4	↑ "1000"	36 cows days
	1000	36
	500	18
		90

$$6 \text{ cows} \quad 90 \text{ cows days}$$

$$90 \div 6 = 15$$

$$? \quad 15$$

$$9 \text{ days} \times 2.5 \times 2 = 45$$

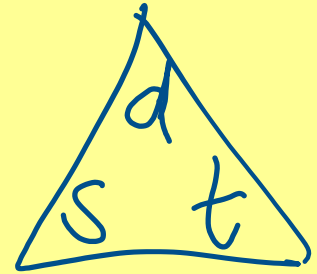
$$= 15$$

Proportional relationships

- I cycle uphill at an average speed of 3 metres per second for 5 minutes. $d = 3 \times 300 = 900m$
- I cycle downhill at an average speed of 7.5 metres per second for 10 minutes. $d = 7.5 \times 600 = 4500m$
- What is my average speed overall?

$$\frac{900m}{300s} + \frac{4500m}{600s}$$

$$= 6ms^{-1}$$



$$\frac{900}{300} + \frac{4500}{600} = \frac{3 + 7.5}{1} = 10.5 \text{ ms}^{-1}$$

Proportional relationships and % change

- I cycle at an average speed of v metres per second for t minutes.
- I then cycle 25% faster for 20% shorter period of time.
- Do I cycle a longer, shorter or the same distance?

$$d = v \times t$$

$$d = vt$$

$$\frac{60t \times 1.25}{\cancel{60t} \times 1.25} = \frac{4}{5} \times \frac{5}{4} = 1$$