GCSE 7+ Session 3 Solutions
Independent Practice
Fluency with quadratic expressions

KING'S MATHS SCHOOL

Revise, refresh, recall *the core knowledge and skills:*

When expanding products of two brackets you need to make sure that ***every*** *term in the first bracket is multiplied by* ***every*** *term in the second bracket, before you simplify by collecting any like terms.*

1)

a) $(2x+1)(2x-3)$

$\equiv (2x)^2 - 3(2x) + 1(2x) - 3$

$\equiv 4x^2 - 6x + 2x - 3 \equiv 4x^2 - 4x - 3$

So, $(2x+1)(2x-3) \equiv 4x^2 - 4x - 3$.

b) $(2x+1)(2x-3)(x-2)$

$\equiv (4x^2 - 4x - 3)(x-2)$ This is the answer from 1a.

$\equiv 4x^3 - 8x^2 - 4x^2 + 8x - 3x + 6$ Multiply each term in 1st bracket by each term in 2nd bracket.

$\equiv 4x^3 - 12x^2 + 5x + 6$ Simplify by collecting like terms.

2)

a) $(3x-4y)^2 \equiv (3x-4y)(3x-4y)$

$\equiv 9x^2 - 12xy - 12xy + 16y^2$

$\equiv 9x^2 - 24xy + 16y^2$

b) $(3x+4y)(3x-4y)$

$\equiv 9x^2 - 12xy + 12xy - 16y^2$

$\equiv 9x^2 - 16y^2$

3)

a) You can factorise $x^2 + 3x - 10$, by finding two numbers that:

- Add to equal 3
- Multiply to equal -10
- 5 and -2 satisfy these.

So, $x^2 + 3x - 10 \equiv (x+5)(x-2)$

b) You can factorise $5x^2 - x - 6$ by finding two numbers that:

- Add to equal -1
- Multiply to equal -30 *-30 is the product of the coefficient of x^2 and the constant term.*
- 5 and -6 satisfy these.
- So replace, $-x$ with $+5x - 6x$ in the expression:

$5x^2 - x - 6$

$\equiv 5x^2 + 5x - 6x - 6$

$\equiv 5x(x + 1) - 6(x + 1)$ Factorise the first and second 'half' of the expression separately.

$\equiv (5x - 6)(x + 1)$ Factorise by $(x + 1)$

4)

a) $x^2 - 25 \equiv (x + 5)(x - 5)$ This is **the difference of two squares**.

b) $2x^3 - 50x \equiv 2x(x^2 - 25)$ $2x$ is the HCF of $2x^3$ and $50x$.

$\equiv 2x(x + 5)(x - 5)$ Again, **the difference of two squares**.

5)

a) $\frac{x}{x^2-9x} \equiv \frac{x}{x(x-9)} \equiv \frac{1}{x-9}$ First **factorise** numerator and denominator

b) $\frac{3x-9}{x^2-9} \equiv \frac{3(x-3)}{(x-3)(x+3)} \equiv \frac{3}{x+3}$ First **factorise** numerator and denominator

c) $\frac{x^2+3x}{x^2-9} \equiv \frac{x(x+3)}{(x-3)(x+3)} \equiv \frac{x}{x-3}$ First **factorise** numerator and denominator

6) a) $x = 1 + \frac{56}{x}$

Multiply **each term** by x

$x^2 = x + 56$

So $0 = x^2 - x - 56$

So $0 = (x - 8)(x + 7)$.

So, $x = 8$ or $x = -7$

b) $(3x - 1)(2x + 1) = 20x + 19$

So $6x^2 + 3x - 2x - 1 = 20x + 19$

So $6x^2 + x - 1 = 20x + 19$

So $6x^2 - 19x - 20 = 0$

To factorise this quadratic, we can look for two numbers that:

- Add to equal -19
- Multiply to equal -120 *-120 is the product of the coefficient of x^2 and the constant term.*
- 5 and -24 satisfy these.
- Replace, $-19x$ with $+5x - 24x$ in the expression:

$6x^2 - 19x - 20$

$= 6x^2 - 24x + 5x - 20$

$= 6x(x - 4) + 5(x - 4)$

$= (6x + 5)(x - 4)$

$(6x + 5)(x - 4) = 0$

So, $x - 4 = 0$ so $x = 4$

or, $6x + 5 = 0$ so $x = -\frac{5}{6}$

Quadratic Formula

Solve $48x^2 + 22x - 15 = 0$ using the quadratic formula.

$$x = \frac{-22 \pm \sqrt{22^2 - 4(48)(-15)}}{2 \times 48} = \frac{-22 \pm \sqrt{3364}}{96} = \frac{-22 \pm 58}{96}$$

So, $x = \frac{-22+58}{96} = \frac{36}{96} = \frac{3}{8}$

or $x = \frac{-22-58}{96} = \frac{-80}{96} = \frac{5}{6}$

Completing the Square

Solve $x^2 + 4x - 4 = 0$ by completing the square.

$0 = x^2 + 4x - 4$

$0 = (x + 2)^2 - 8$

So, $(x + 2)^2 = 8$

$x + 2 = \pm\sqrt{8} = \pm 2\sqrt{2}$ Remember **both** positive **and** negative square roots.

So $x = -2 \pm 2\sqrt{2}$